Basic Probability rules

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Axioms of probability

1. The probability $P(A)$ of any event A satisfies $0 \leq P(A) \leq 1$
2. If **S** is the sample space, then $P(S) = 1$
3. If A_1, A_2, A_3 ... are pairwise disjoint, then

$$P(A_1 \cup A_2 \cup A_3 \ldots) = P(A_1) + P(A_2) + \mathrm{P}(\mathrm{A}_3) + \ \ldots$$

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Complement rule

The probability that an event occurs and does not occur always add to 1:

$$P(A) + P(A^c) = 1$$

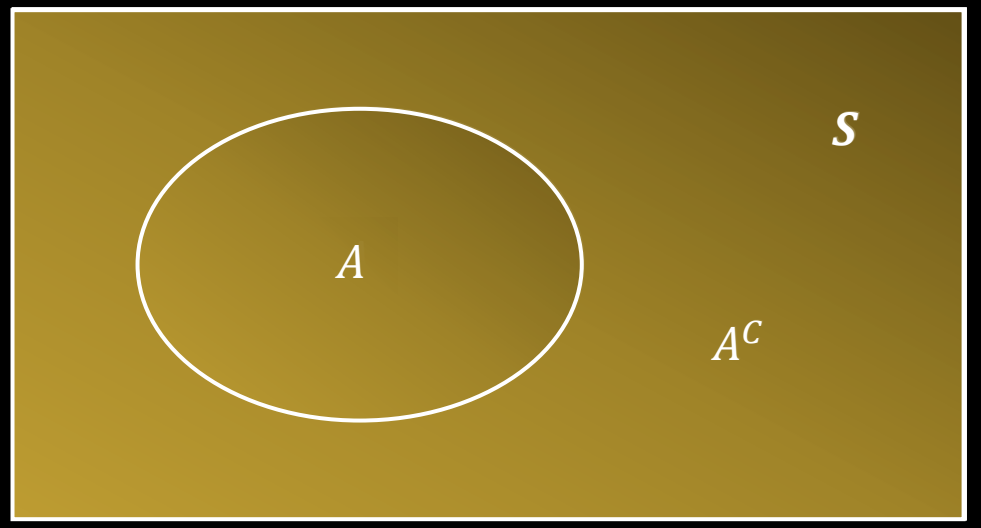
$$\Rightarrow P(A^c) = 1 - P(A)$$

$$\Rightarrow P(A) = 1 - P(A^c)$$

Proof :

$$1 \overset{2.axiom.}{=} P(S) = P(A \cup A^c) \overset{3.axiom}{=} P(A) + P(A^c)$$

$S = A \cup A^c$



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Complement rule

Example: A factory produces items, and each item has a 1% defect rate. What is the probability that at least one defective item appears in a batch of 100 items?

$$P(at\ least\ one\ defective)$$
$$= P(one\ defective) + P(two\ defective) + \ ... + P(100\ defective)$$

$$P(at\ least\ one\ defective) = 1\ - P(no\ defective)$$
$$= 1\ - 0.99^{100}\ \approx 0.63$$

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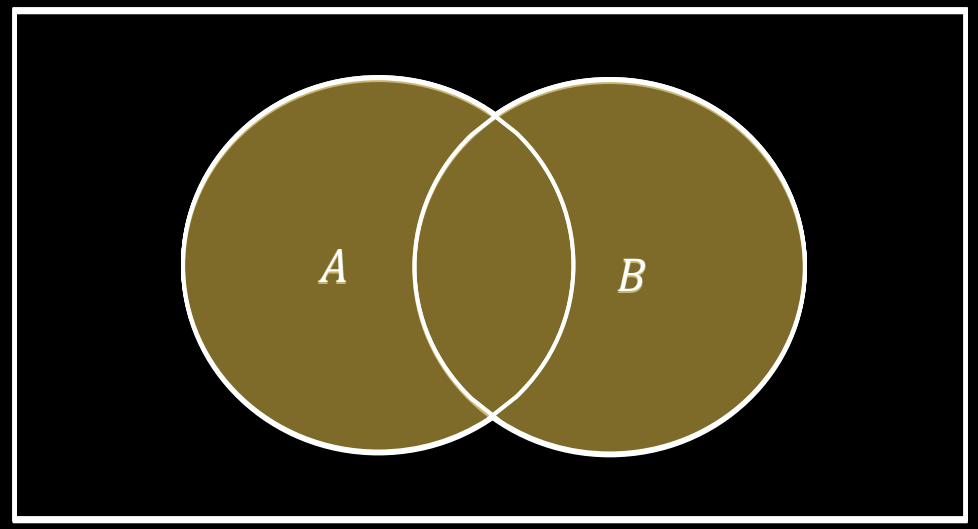
General addition rule for unions of two events

For any two events A and B:

$$P(A \cup B) = P(A) + \mathrm{P(B)} - P(A \cap B)$$



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General addition rule for unions of two events

Example: Drawing a card from a deck.

- A: The event that the card is a **Heart** (13 hearts in the deck)
- B: The event that the card is a **King** (4 kings in the deck)

What is the probability that the card is a heart and/or a king ($A \cup B$)?

$$P(A \cup B) = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A} \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$$

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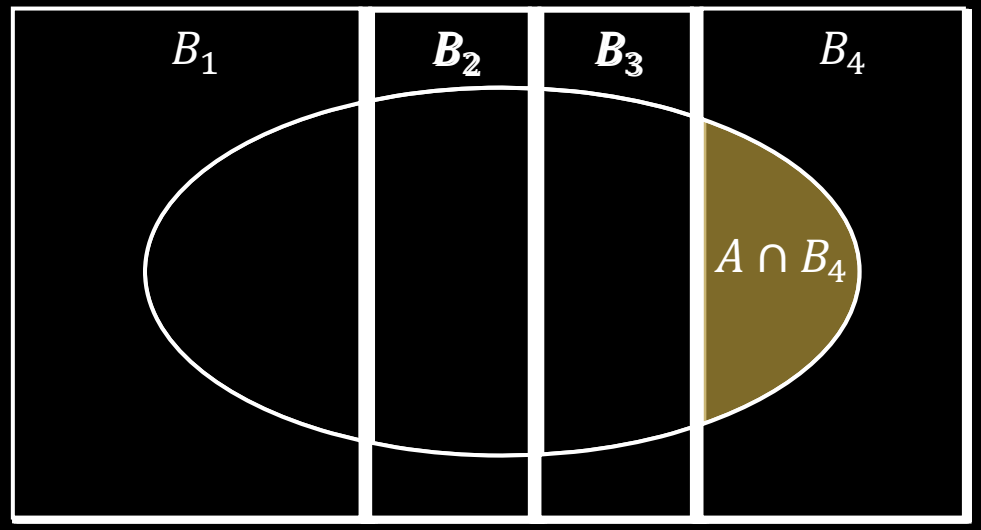
Law of total probability

Assume $B_1, B_2, \ldots, B_k$ are disjoint events that divide up the whole sample space so that their probabilities add to exactly 1. Then, if A is any other event

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ \ldots + P(A \cap B_k)$$



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Law of total probability

Example: Drawing a card from a deck.

- B_1: The event that the card is a **Heart ♥**
- B_2: The event that the card is a **Diamond ♦**
- B_3: The event that the card is a **Spade ♠**
- B_4: The event that the card is a **Club ♣**

What is the probability of the event A: that the card is a king?

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + P(A \cap B_4)$$

$$= P(\heartsuit K) + P(\diamondsuit K) + P(\spadesuit K) + P(\clubsuit K)$$

$$= \frac{1}{52} + \frac{1}{52} + \frac{1}{52} + \frac{1}{52} = \frac{4}{52}$$

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Summary

1. Range: $0 \leq P(A) \leq 1$
2. Something will happen: $P(S) = 1$
3. Union of disjoint events: $P(A \cup B) = P(A) + P(B)$
4. Complement rule: $P(A) + P(A^c) = 1$
5. General rule for unions: $P(A \cup B) = P(A) + P(B) - \mathrm{P}(\mathrm{A} \cap B)$
6. Law of total probability: $P(A) = P(A \cap B_1) + P(A \cap B_2) + \ \ldots + P(A \cap B_k)$

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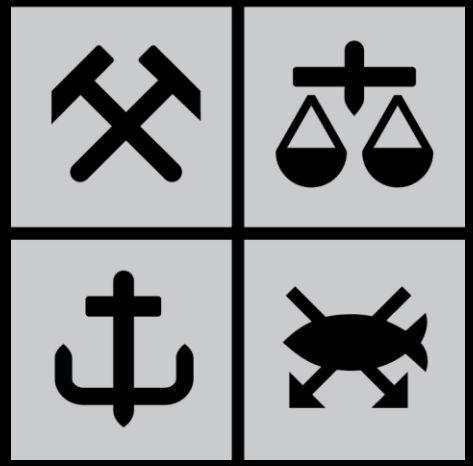

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