

# THE BINOMIAL DISTRIBUTION

The number of **successes** in  ***$n$***  **independent Bernoulli trials** has a binomial distribution

If  $X$  is the number of successes in  $n$  trials, each with success probability  $p$ , then

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

*for  $x \in \{0, 1, 2, \dots, n\}$*

## Scenario:

A company knows from past campaigns that 10% of customers click a marketing email.

They send an email to  $n = 10$  customers.

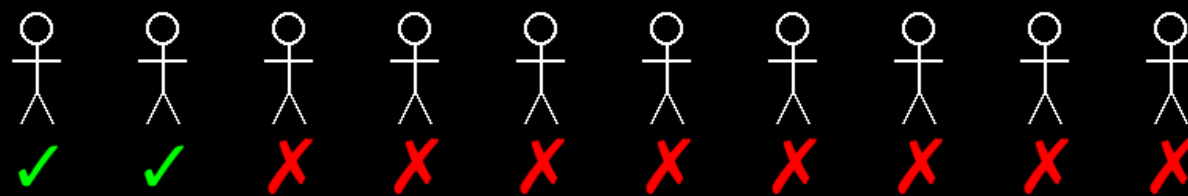
## Question:

What is the probability that exactly 2 customers click?

$$P(\text{stick figures with } \checkmark \text{ and } \times)$$

$$= 0.1 \cdot 0.1 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$

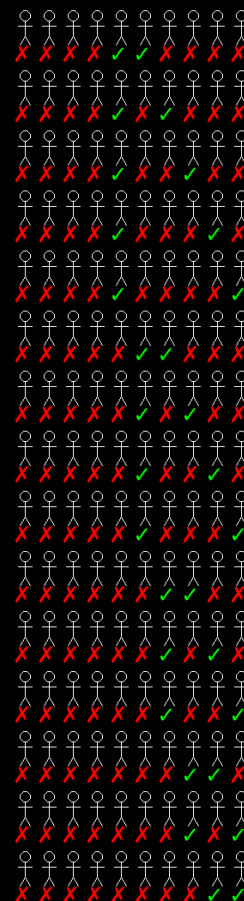
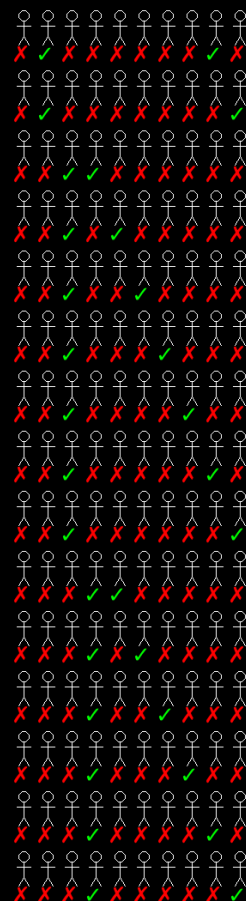
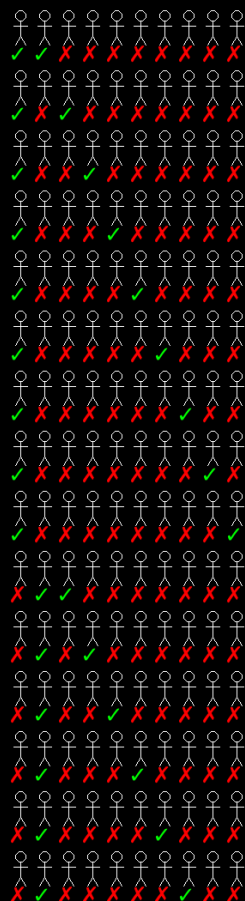


$$P(\text{stick figures with checkmarks and crosses})$$

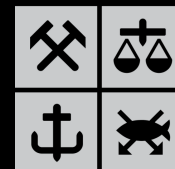
$$= 0.1 \cdot 0.9 \cdot 0.1 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$

45



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$$P(\text{Two clicks}) = P\left( \begin{array}{c} \text{Grid 1} \\ \cup \\ \text{Grid 2} \\ \cup \\ \text{Grid 3} \end{array} \right)$$

The equation illustrates the probability of two clicks occurring, represented as the union of three distinct sets of outcomes. Each set is visualized as a 10x10 grid of stick figures. In each grid, 10 figures have a green checkmark (indicating a 'click') and 90 figures have a red 'X' (indicating no click). The positions of the green checkmarks vary across the three grids, representing different scenarios or trials.

$$\begin{aligned} P(\text{Two clicks}) &= \overbrace{0.1^2 0.9^8 + 0.1^2 0.9^8 + \dots + 0.1^2 0.9^8}^{45} \\ &= 45 \times 0.1^2 0.9^8 \approx 0.19 \end{aligned}$$


$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

where  $X$  follows a binomial distribution with  $n = 10$  trials, and success probability  $p = 0.1$ .

$$X \sim \text{Bin}(10, 0.1)$$


$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8$$

«10 choose 2»



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$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8 = 45 \cdot 0.1^2 0.9^8 \approx 0.19$$

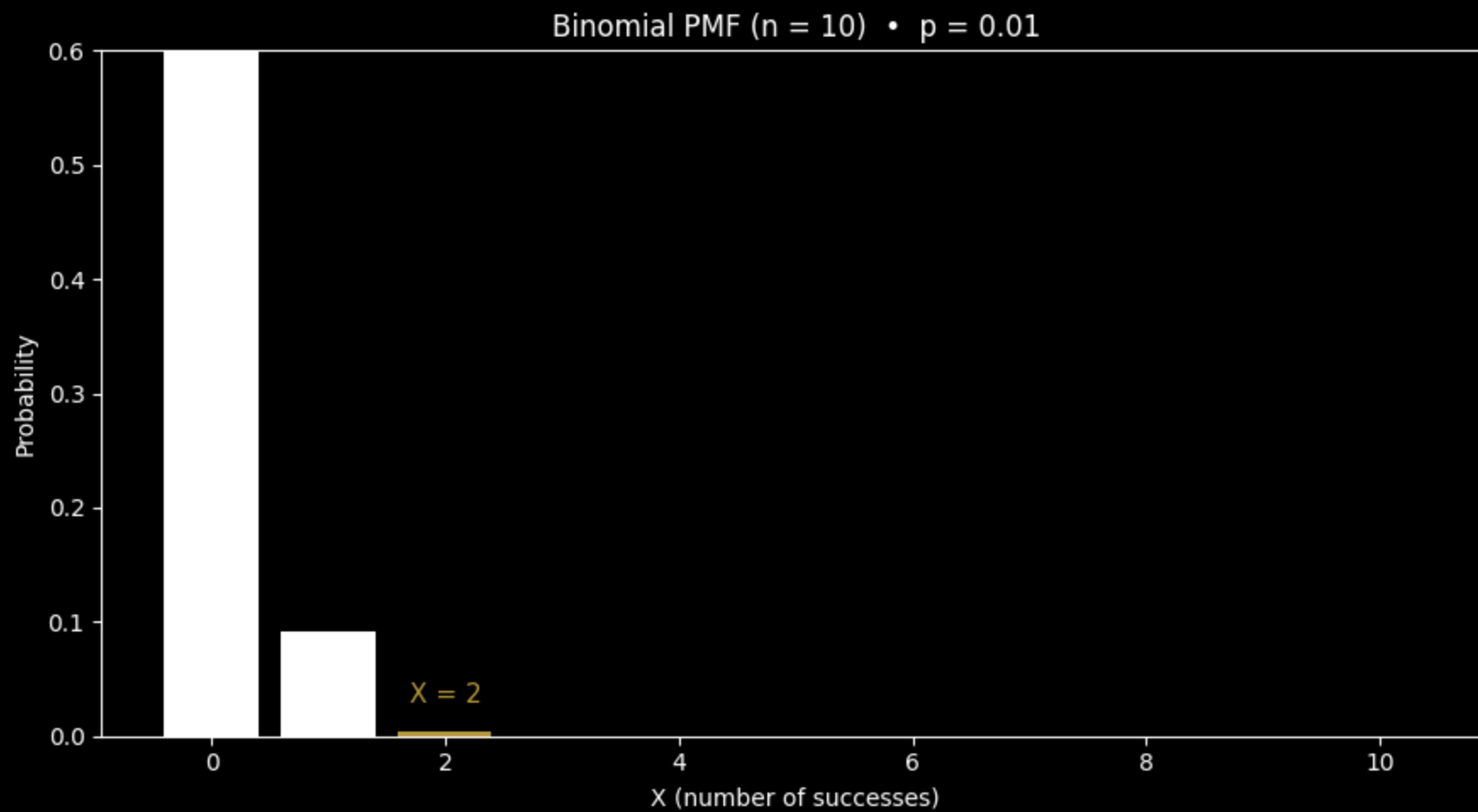
«10 choose 2»  $\binom{10}{2} = \frac{10!}{2!(10-2)!} = 45$

$$P(\text{✓ ✗ ✓ ✗ ✗ ✗ ✗ ✗ ✗ ✗})$$

Number of ways to choose  $x$  successes in  $n$  trials

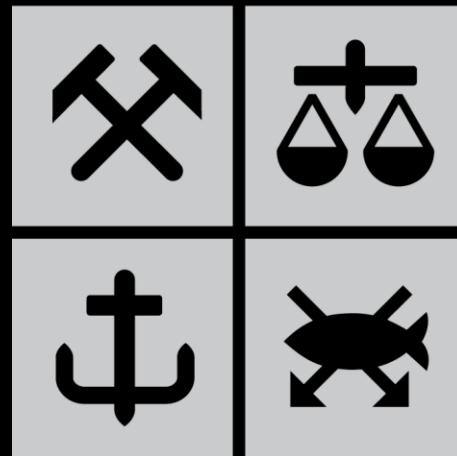
Probability of any specific sequence with  $x$  successes

$$P(X = x) = \binom{n}{x} \underbrace{p^x (1 - p)^{n-x}}$$





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