The Binomial Distribution

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The number of **successes** in n **independent Bernoulli trials** has a binomial distribution

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If X is the number of successes in n trials, each with success probability p, then

$$\mathrm{P}(\mathrm{X} = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$for\ x \in \{0,1,2, \dots, n\}$$

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Scenario:

A company knows from past campaigns that 10% of customers click a marketing email.

They send an email to n = 10 customers.

Question:

What is the probability that exactly 2 customers click?

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$$P(\text{✓ ✓ ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗})$$

$$= 0.1 \cdot 0.1 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$

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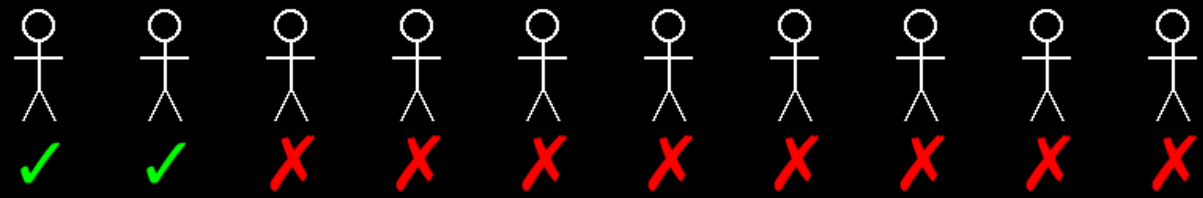

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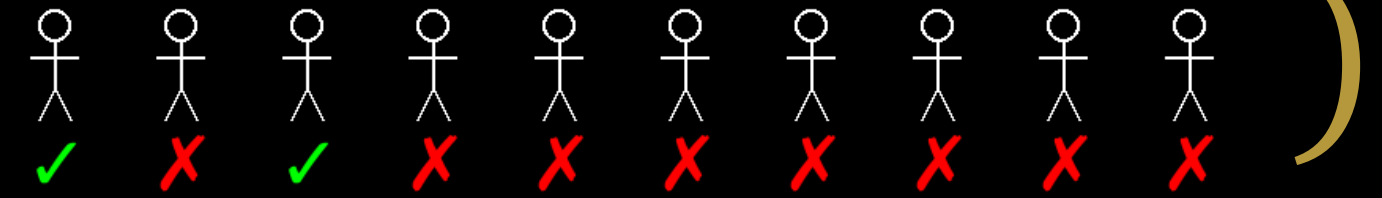
$$P(\text{  })$$

$$= 0.1 \cdot 0.9 \cdot 0.1 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$$

$$= 0.1^2 0.9^8$$

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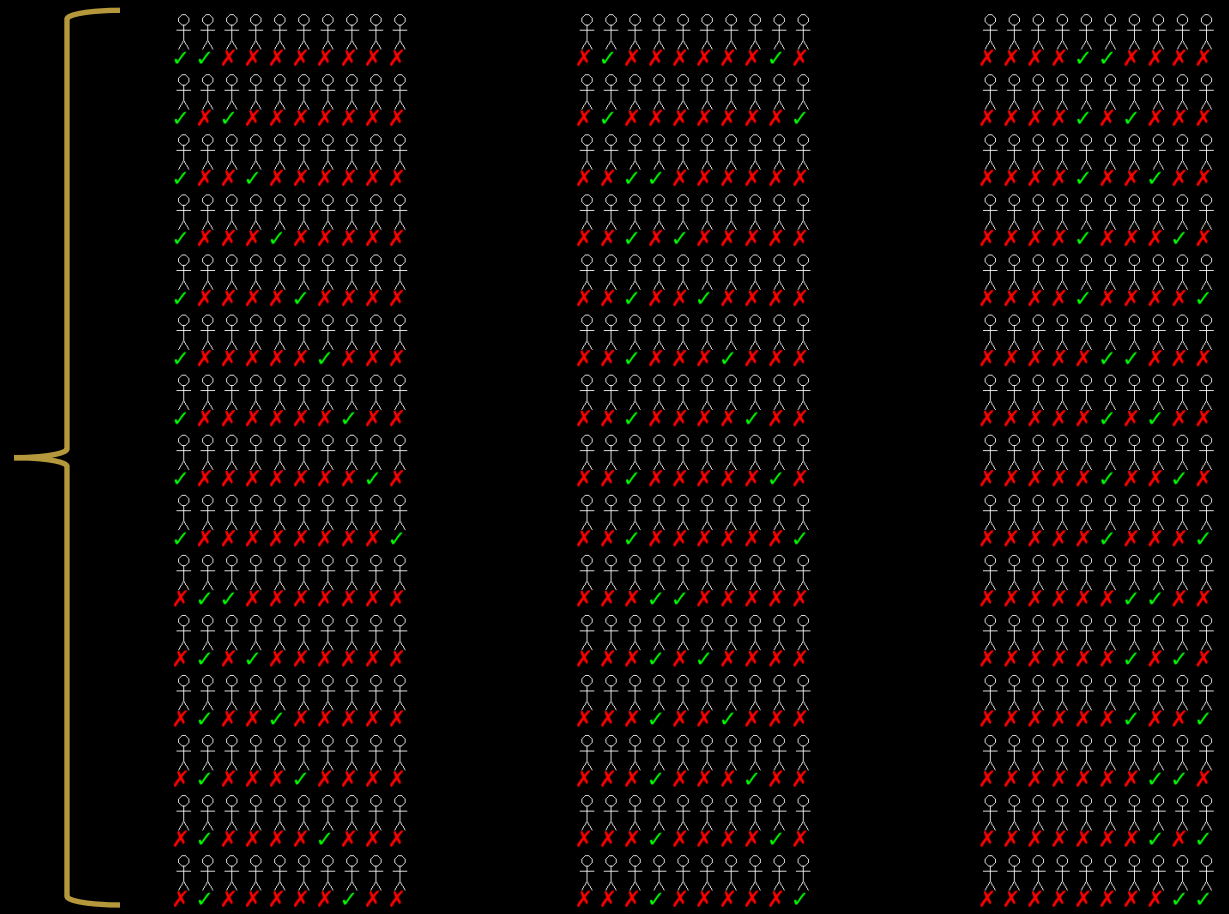
45



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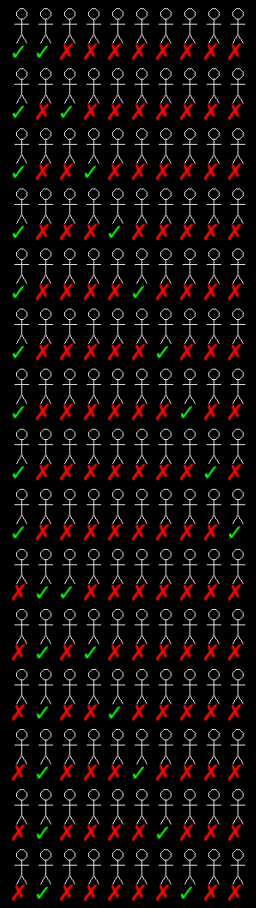
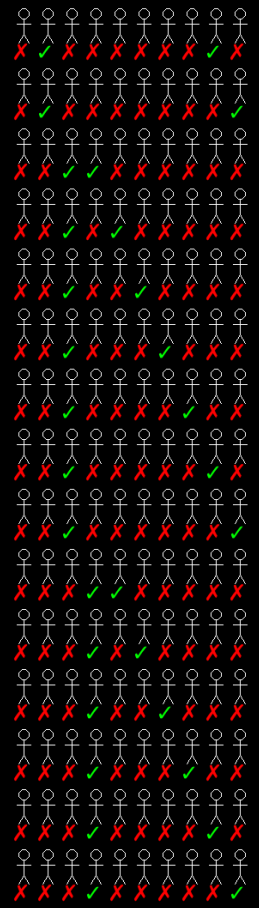
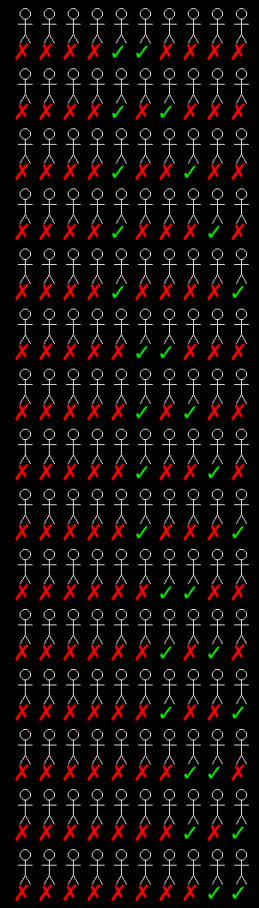
$$P(\text{Two clicks}) = P($$  $\cup$  $\cup$  $)$

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$$P(Two\ clicks) = \overbrace{0.1^2 0.9^8 + 0.1^2 0.9^8 + \cdots + 0.1^2 0.9^8}^{45}$$

$$= 45 \times 0.1^2 0.9^8 \approx 0.19$$

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$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where X follows a binomial distribution with $n = 10$ trials, and success probability $p = 0.1$.

$$X \sim Bin\,(10, 0.1)$$

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«10 choose 2»

$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8$$

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«10 choose 2»

$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8$$

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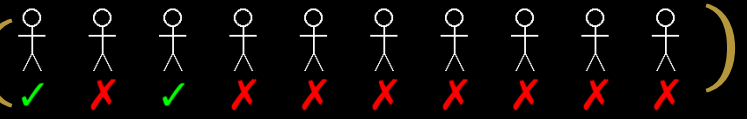
«10 choose 2» $\binom{10}{2} = \frac{10!}{2!\,(10-2)!} = 45$

$$P(X = 2) = \binom{10}{2} 0.1^2 0.9^8 = 45 \cdot 0.1^2 0.9^8 \approx 0.19$$

$P($  $)$

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Number of ways to choose x successes in n trials

Probability of any specific sequence with x successes

$$\mathrm{P}(\mathrm{X} = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

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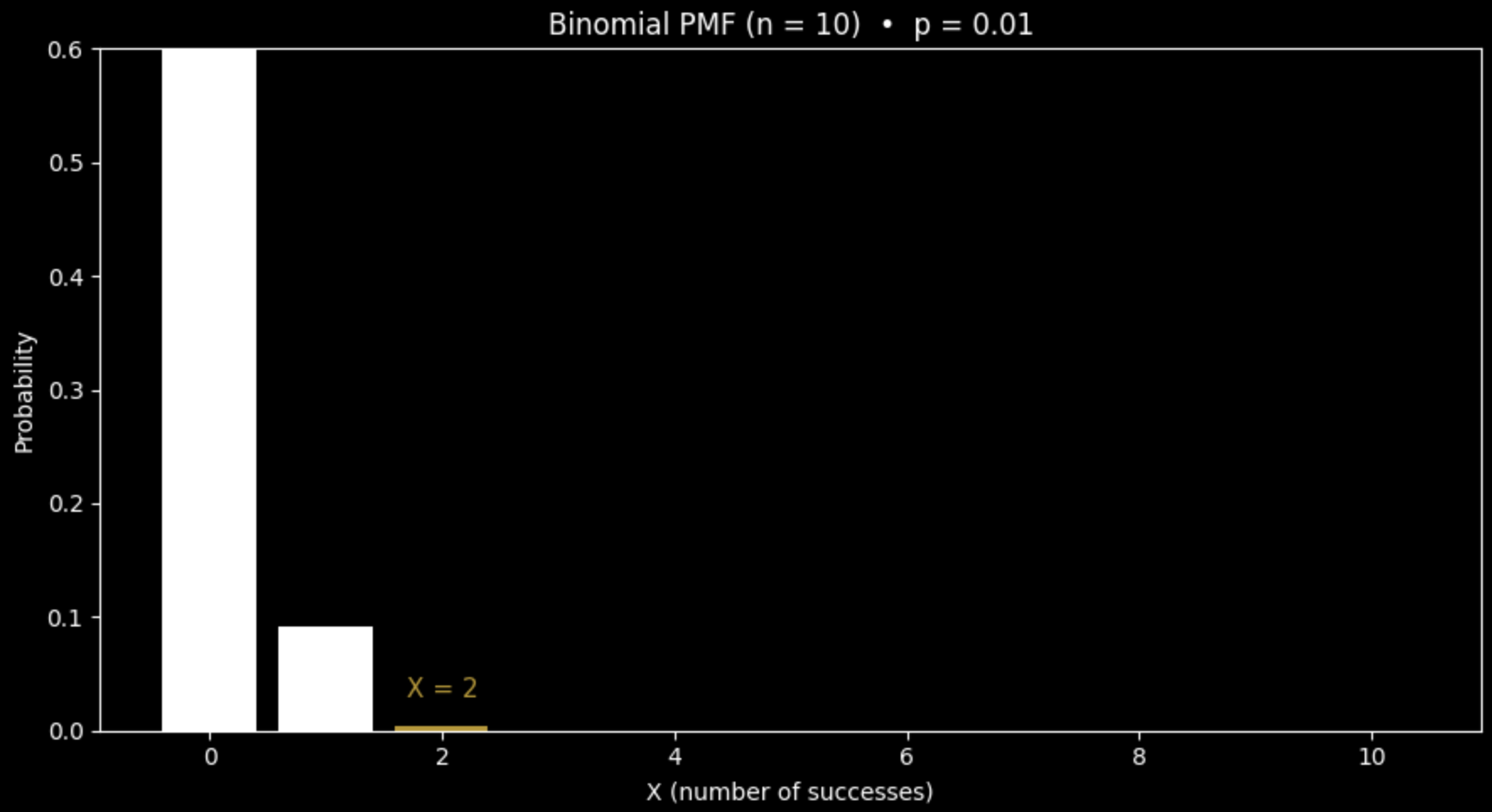
Binomial PMF (n = 10) • p = 0.01
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
X = 2
0
2
4
6
8
10
X (number of successes)

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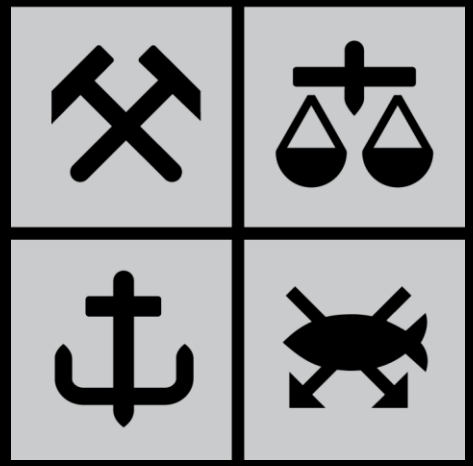

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