Independent events

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DEFINITION

Two events A and B are independent if and only if:

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\cancel{P(A)P(B)}}{\cancel{P(B)}} = P(A) \quad \text{For } P(B) > 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\cancel{P(A)}P(B)}{\cancel{P(A)}} = P(B) \quad \text{For } P(A) > 0$$

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Equivalent statements

For two events A and B with positive probability of occurring, the following 3 statements are equivalent:

1. $P(A \cap B) = P(A)P(B)$
2. $P(A|B) = P(A)$
3. $P(B|A) = P(B)$

→ **Two events are independent if knowing that one occurs does not change the probability that the other occurs.**

Either they are all true and the events A and B are independent.

Or they are all false and the events A and B are not independent.

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Example: We randomly select a card from a deck of 52 ordinary cards

Are the following events independent? $\mathrm{P(A) = P(A|B)?}$

A: The card is spade
B: The card is a 5

$$\mathrm{P(A)} = \frac{13}{52} = \frac{1}{4}$$

$$\mathrm{P(A|B)} = \frac{1}{4}$$

$\mathrm{P(A) = P(A|B)}$, so A and B are independent.

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Example: We randomly select a card from deck of 52 ordinary cards

Are the following events independent? $\mathrm{P(A)} = \mathrm{P(A|B)}?$

A: The card is a facecard

B: The card is a 5

$$\mathrm{P(A)} = \frac{12}{52}$$

$$\mathrm{P(A|B)} = \frac{0}{4} = 0$$

$\mathrm{P(A)} \neq \mathrm{P(A|B)}$, so A and B are not independent.

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Example: Every second customers in a store are shown a friendly reminder (a nudge) to buy reusable shopping bags.

The results from 100 customers were the following:

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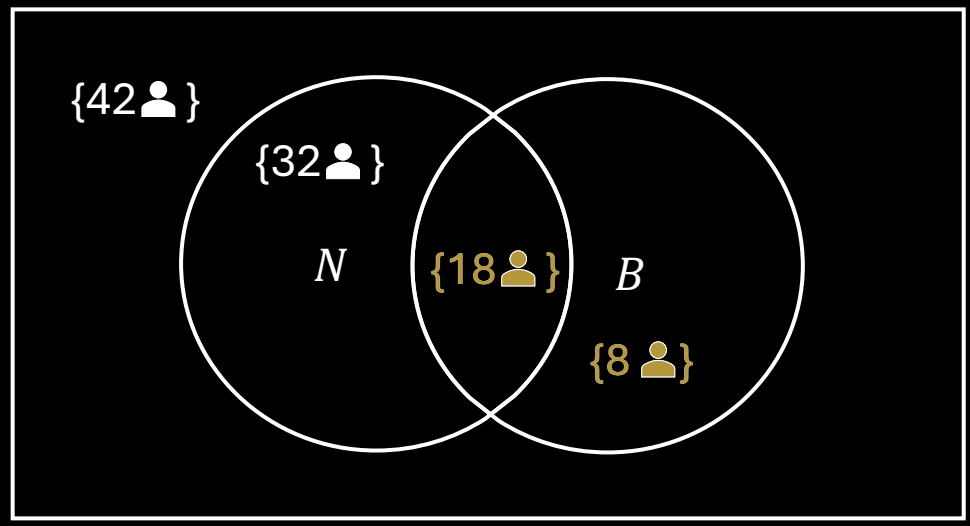
	Nudge: No	Nudge: Yes	Total
Reusable Bag: No	42	32	**74**
Reusable Bag: Yes	8	18	**26**
Total	50	50	**100**

Suppose one of these customers is randomly selected.

Are the following events independent?

B: The customer bought a reusable bag
N: The customer was nudged

$$P(B) = \frac{26}{100} = 0.26$$



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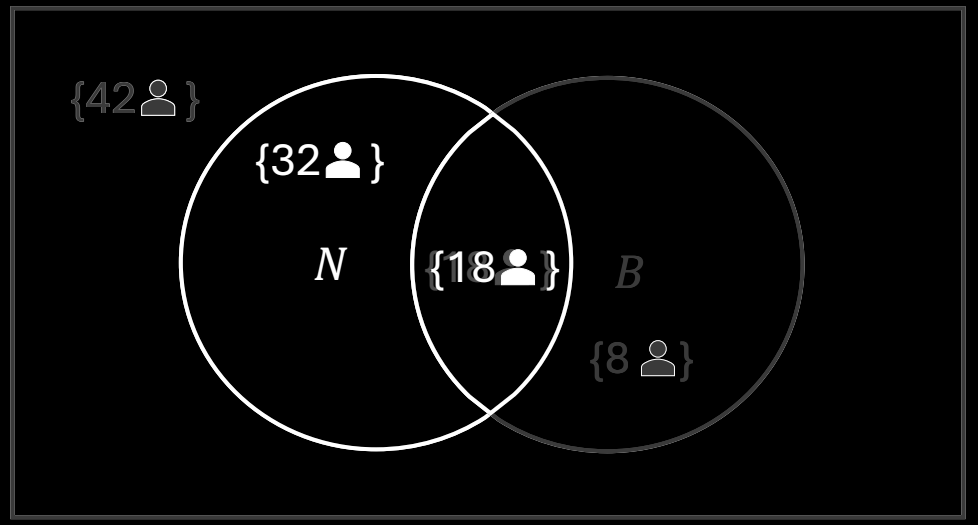
	Nudge: No	Nudge: Yes	Total
Reusable Bag: No	42	32	74
Reusable Bag: Yes	8	18	26
Total	50	50	100

$$P(B|N) = \frac{18}{50} = 0.36$$

$P(B) < P(B|N)$, so *B* and *N* are not independent.



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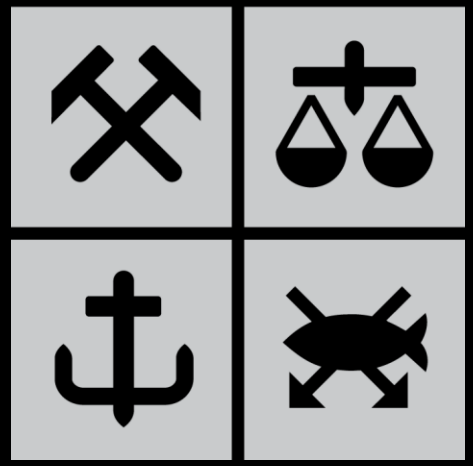

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