Bayes' rule

Example: Fraud Detection in Credit Card Transactions

Imagine a bank is using an algorithm to detect potential credit card fraud.

- Let *F represent a fraudulent transaction*
- *Let T represent a transaction flagged as fraudulent*

We know the so called ***Sensitivity*** $P(T|F)$: The probability that the algorithm flags a transaction as fraudulent given that it is actually fraudulent.

However, what the bank (and its customers) really want to know is:

What is the probability that a transaction is actually fraudulent, given that the algorithm has flagged it as potentially fraudulent?

$$P(F|T)$$

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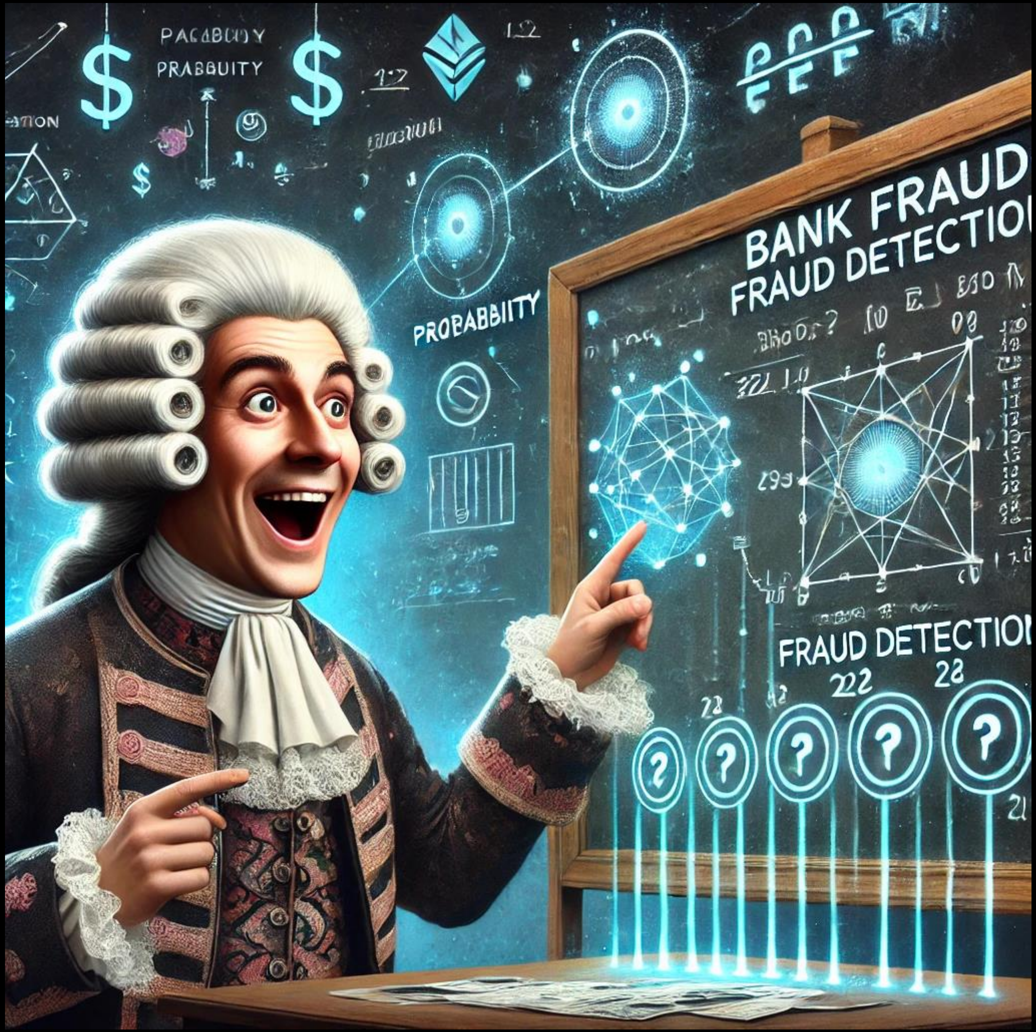
$P(B|A)$ known, but want to know $P(A|B)$?

Then we need Bayes' rule!



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Bayes rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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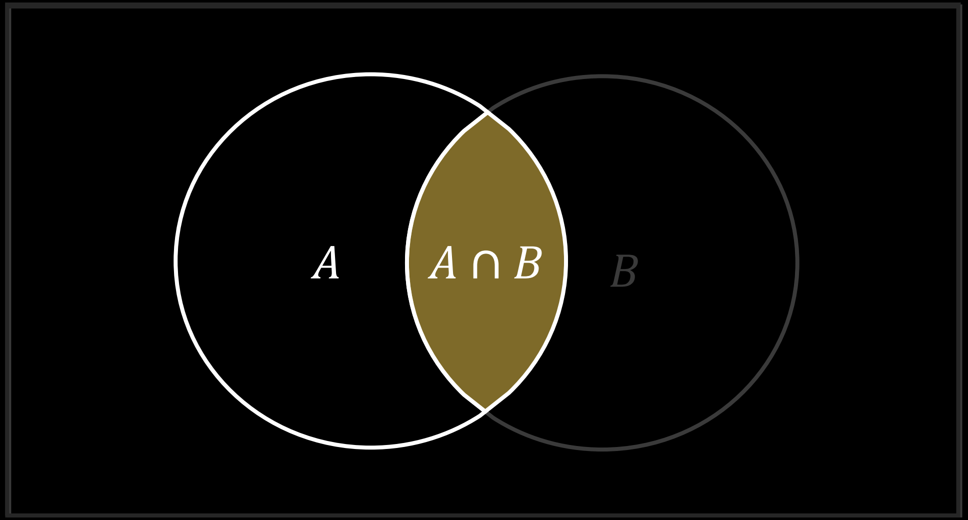
PROOF

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$



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A TRICKY DENOMINATOR

$$P(A|B) = \frac{P(B|A)P(A)}{\boxed{P(B)}}$$

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3 versions of Law of total probability

Assume $A_1, A_2, \dots, A_k$ are disjoint events that divide up the whole sample space so that their probabilities add to exactly 1. Then, if B is any other event

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_k \cap B)$$

$$= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots + P(B|A_k)P(A_k)$$

Special case: A and A^c are examples of disjoint events dividing up the whole sample space:

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

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3 versions of Bayes' rule

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$
$$= P(B|A_1)P(A_1) + \cdots + P(B|A_k)P(A_k)$$

1. $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

2. $P(A|B) = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^C)P(A^C)}$

3. $P(A_j|B) = \dfrac{P(B|A_j)P(A_j)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_k)P(A_k)}$

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Example: Fraud Detection in Credit Card Transactions

We know:

- $P(T|F) = 0.90$ *(sensitivity)*
- $P(F) = 0.01$ *(base rate of fraud)*
- $P(T|F^c) = 0.05$ *(false positive rate)*
- $P(F^c) = 0.99$ *(base rate of legitimate transactions)*

We can then use the following version of Bayes rule:

$$P(F|T) = \frac{P(T|F)P(F)}{P(T|F)P(F) + P(T|F^c)P(F^c)} = \frac{\cdot}{\cdot + \cdot} \approx 0.15$$

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A TYPICAL MISTAKE

Base rate fallacy: It is easy to overestimate the likelihood of fraud when a transaction is flagged because we focus on the high sensitivity and low false positives of the algorithm, neglecting the very low base rate of fraud P(F).

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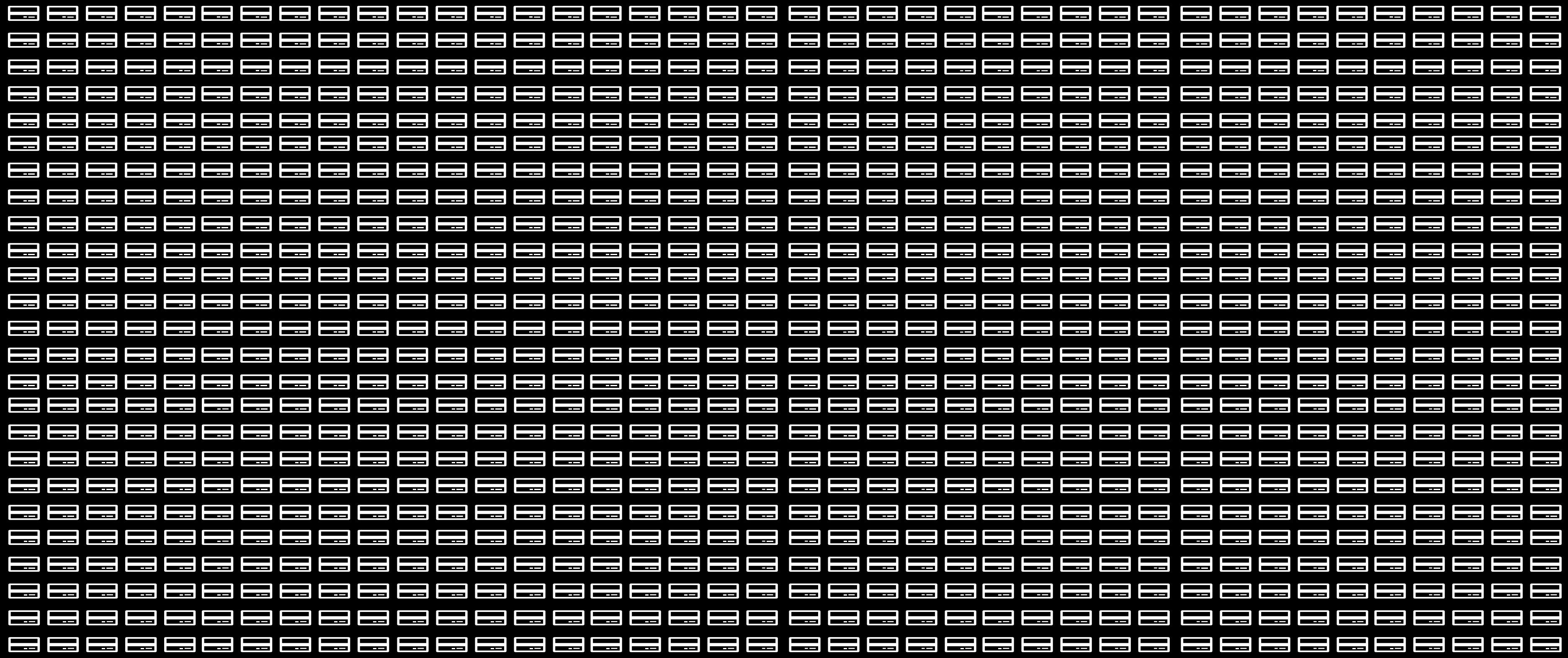
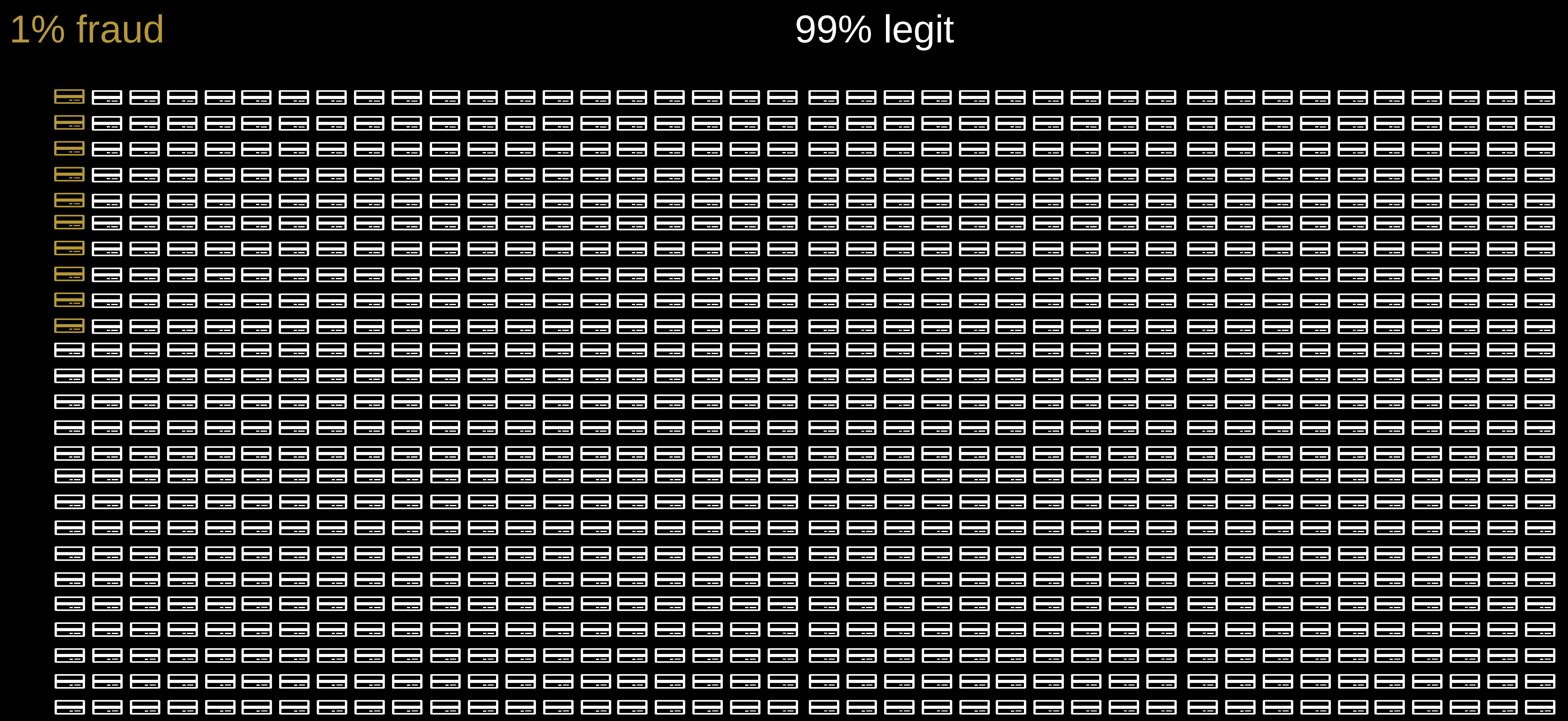
Base rate fallacy

Sample of 1000 transactions:





- $P(T|F) = 0.90$ *(sensitivity)*

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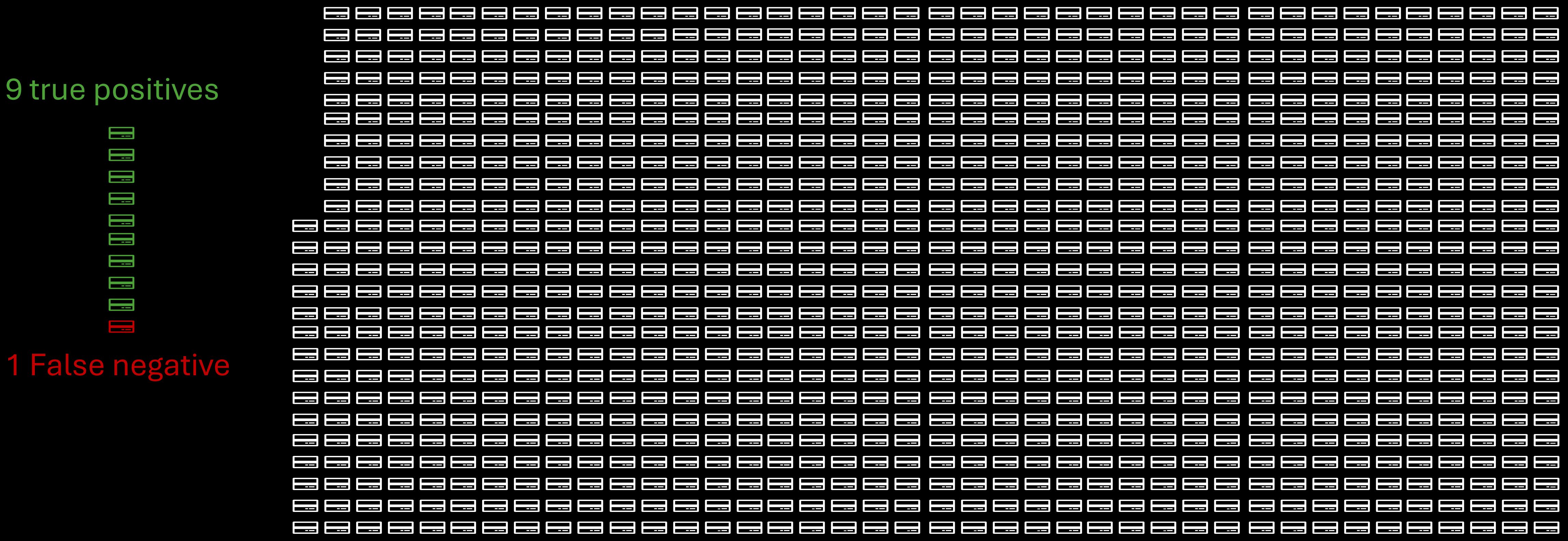

- $P(T|F^c) = 0.05$ *(false positive rate)*

- $P(T|F) = 0.90$ *(sensitivity)*

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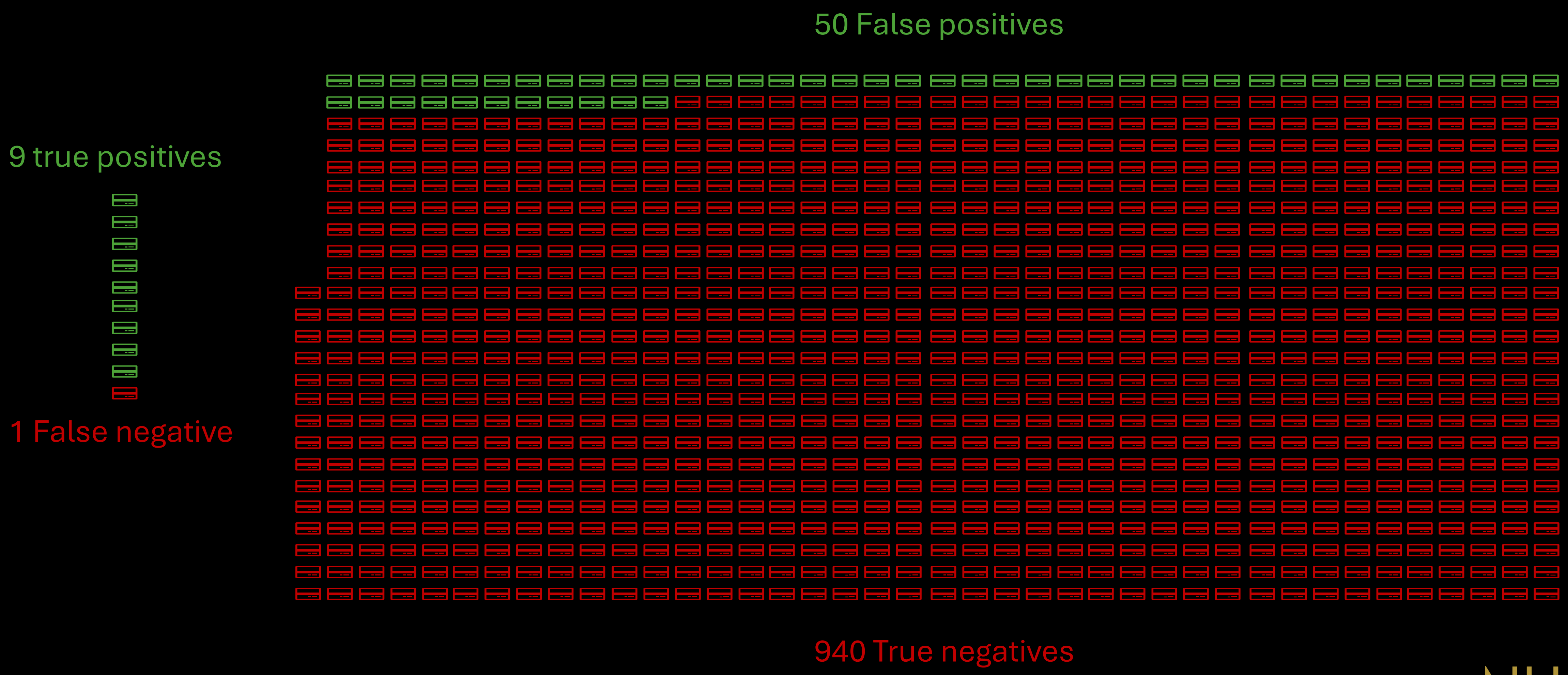
50 False positives
9 true positives
1 False negative
940 True negatives

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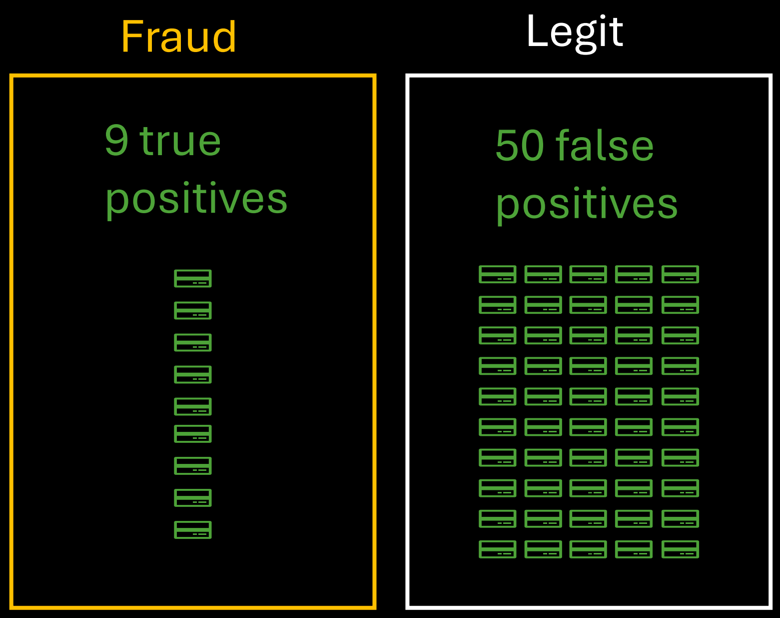

$$P(F|T) \approx \frac{9}{9+50} \approx 0.15$$

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