Conditional probability

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Example: Rolling a dice.

Suppose we are told that the dice came up with an odd number. What is the probability that it is a 5?

$$S = \{1, 2, 3, 4, 5, 6\} \longrightarrow S^{new} = \{1, 3, 5\}$$

$$P(5 \mid Odd) = \frac{1}{3}$$

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DEFINITION

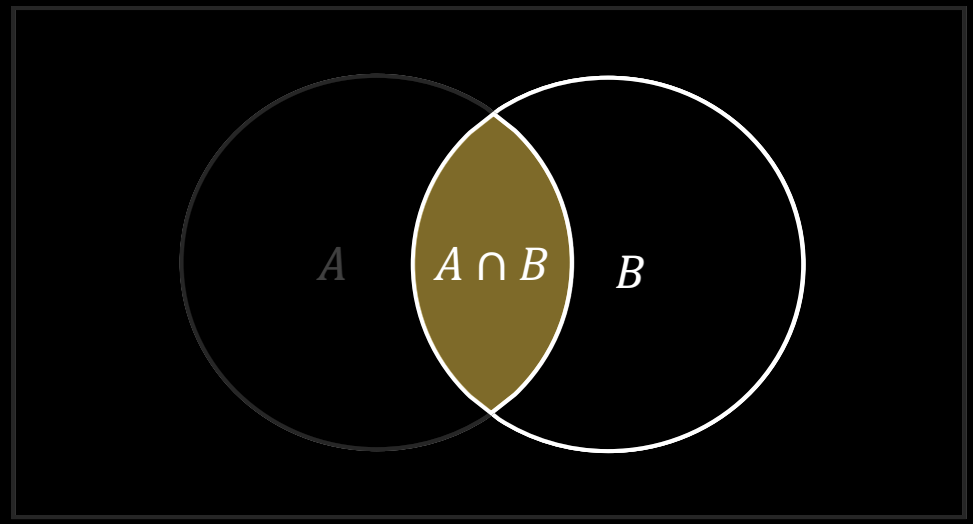
We often wish to determine the probability of some event A given that some other event B has occurred, which are known as *conditional probabilities,* $P(A|B)$.

- The symbol "|" can be read "given"

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Assuming $P(B) > 0$

$$P(B|B) = \frac{P(B \cap B)}{P(B)} = 1$$



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Example: Rolling a dice.

- $A = \{2, 3, 4, 5, 6\}$
- $B = \{1, 3, 5\}$ (odd) ← The reduced sample space

What is the conditional probability of A, given B?

$$P(A|B) = \frac{2}{3} \qquad P(A) = \frac{5}{6}$$

$$A \cap B = \{3,5\}$$

$$\frac{P(A \cap B)}{P(B)} = \frac{2/6}{3/6} = \frac{2}{3}$$

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Example: Consider an experiment where every second customers in a store are shown a friendly reminder (a nudge) to buy reusable shopping bags.

The results from 100 customers were the following:

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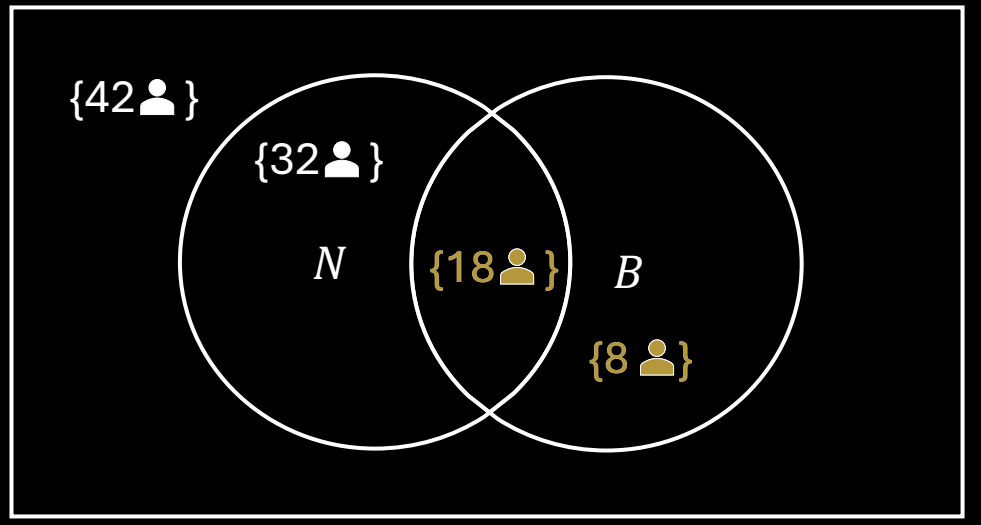
	Nudge: No	Nudge: Yes	Total
Reusable Bag: No	42	32	**74**
Reusable Bag: Yes	8	18	**26**
Total	50	50	**100**

Suppose one of these customers is randomly selected.

What is the probability that the customer bought a reusable bag?

$$P(B) = \frac{26}{100} = 0.26$$



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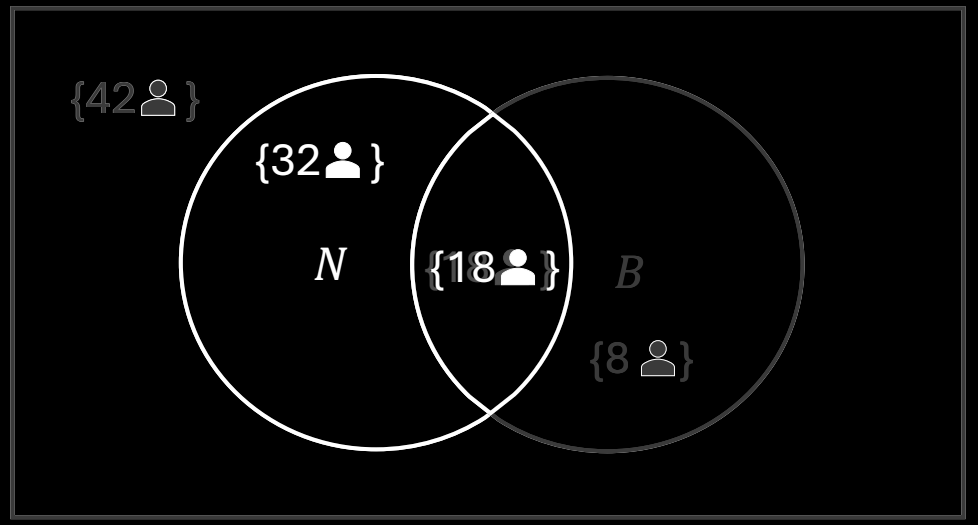
	Nudge: No	Nudge: Yes	Total
Reusable Bag: No	42	32	74
Reusable Bag: Yes	8	18	26
Total	50	50	100

Given that the customer was given a nudge, what is the probability that the customer bought a reusable bag?

$$P(B|N) = \frac{18}{50}$$



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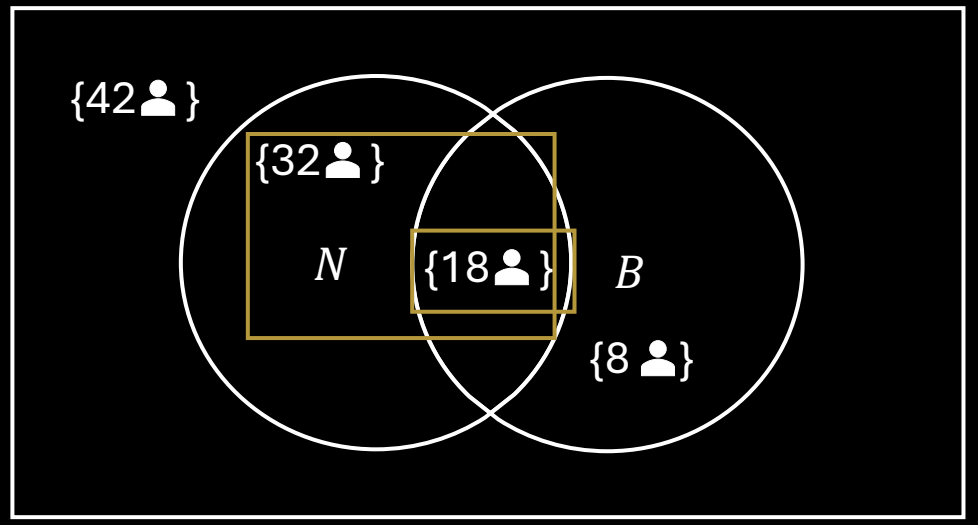
$$P(B|N) = \frac{18}{50}$$

$$P(B) = 0.26$$

$$\frac{P(B \cap N)}{P(N)} = \frac{18/100}{50/100}$$

$$= \frac{18}{50} = 0.36$$



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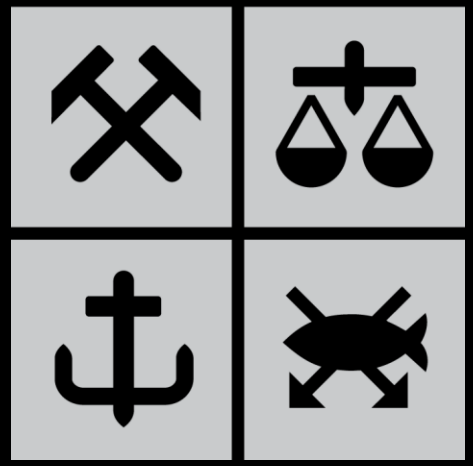

Sondre Hølleland
Geir Drage Berentsen