Discrete random variables and distributions

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Experiment: Opinion poll, in which we ask 50 students if they agree that 3D pie charts are terrible.

Sample space of this experiment is:

{yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes}
{no, no, yes, bo, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, yes, yes, yes, yes, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, yes, yes, yes, no, no, no, no, no, yes}
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{no, no, no, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, no, no, no, yes, no, yes, yes, no, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, no, no, no, no, yes, no, no}

2^{50}

X = Number of «yes» among the 50 students

Sample space for X : {1,2,3,….,50}

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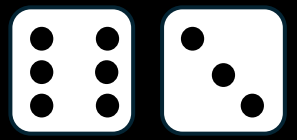
Random variables

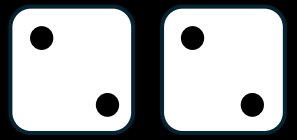
A **random variable** associates a **numerical value** to the **outcomes** of a random experiment.

Example: Rolling a pair of dice.

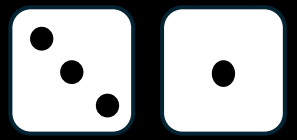
$X = \text{Sum}$



$X = 9$



$X = 4$



$X = 4$

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Experiment: Rolling a pair of dice.

$X = \text{Sum}$

Possible values: $\{2,3,4,5,6,7,8,9,10,11,12\}$

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A **discrete random variable** has possible values that can be given in an ordered list.

- The sum of two dice {2,3,4…,12}
- The number of calls you need to make before successfully connecting to customer service {1,2,3,…}

A **continuous random variable** takes all values in some interval.

- Annual income of a randomly selected person

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Probability distribution

The **probability distribution** of a discrete random variable X is a list of all possible values of X and their probabilities.

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X = Sum of a pair of dice.

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	0.0278	0.0556	0.0833	0.1111	0.1389	0.1667	0.1389	0.1111	0.0833	0.0556	0.0278

$$p(x) = P(X = x)$$

Short notation

Random variable

A value of the random variable

$$P(X = 10) = P(\square\square) + P(\square\square) + P(\square\square)$$

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} \approx 0.0833$$

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All discrete probability distributions **must satisfy**:

1. $0 \leq p(x) \leq 1$ for all x
2. $\sum_{\{all\ x\}} p(x) = 1$
3. Find the probability of any event by adding the probabilities of the values that make up the event.

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	0.0278	0.0556	0.0833	0.1111	0.1389	0.1667	0.1389	0.1111	0.0833	0.0556	0.0278

$$P(X \leq 4) = p(2) + \mathrm{p}(3) + \mathrm{p}(4) = 0.0270 + 0.0556 + 0.0833 = 0.1659$$

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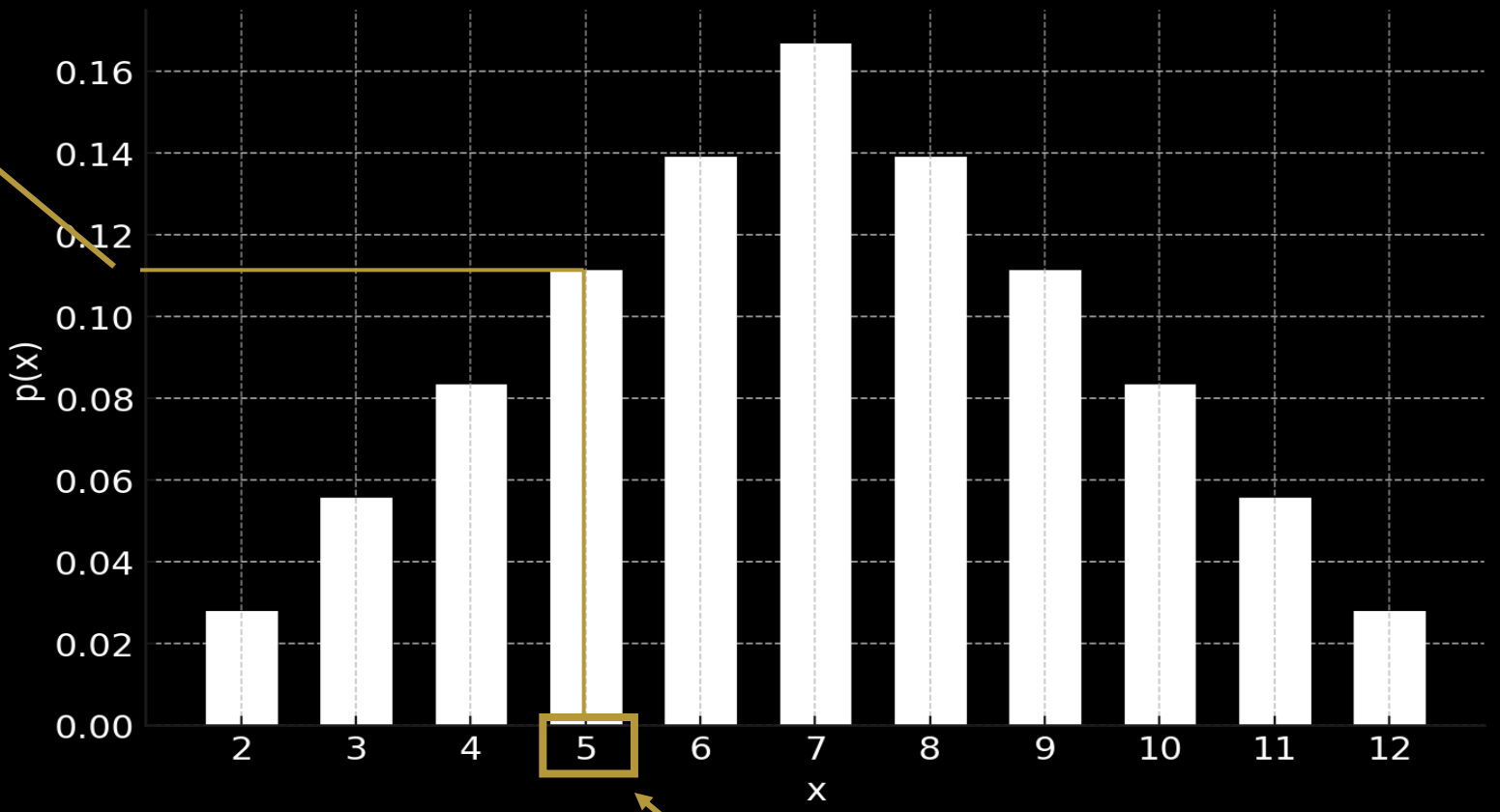
Probability histogram

$P(X = 5) = 0.1111$



$X = 5$

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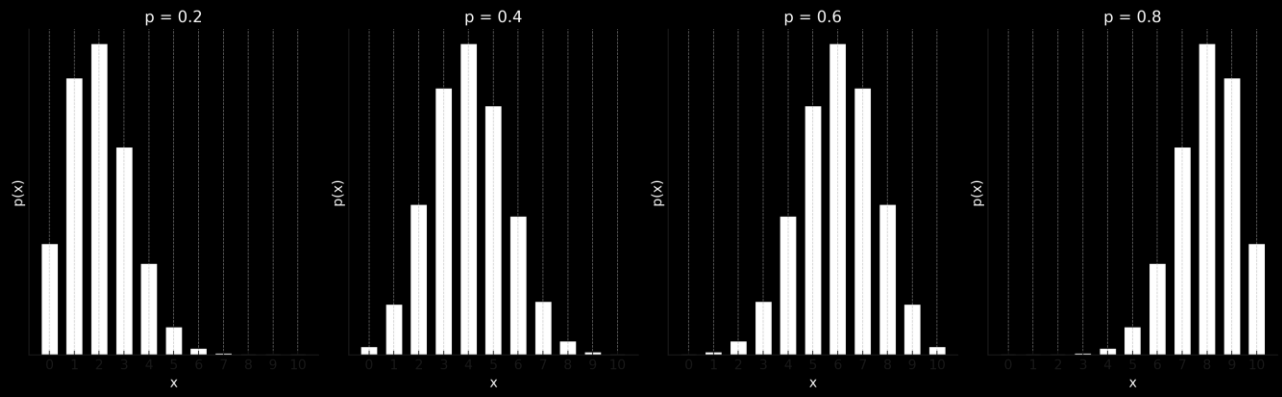
Probability distribution formulas

In some situations the probability distribution can be expressed as a formula.

Example: The binomial distribution

$$p(x) = \binom{n}{x} p^x (1-\mathrm{p})^{n-x} \quad for\ x = 0,1,2,\ldots n$$

parameters



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COMMON DISTRIBUTIONS

Examples of **discrete probability distributions**:

- Binomial
- Poisson
- Geometric
- Negative Binomial
- Hypergeometric

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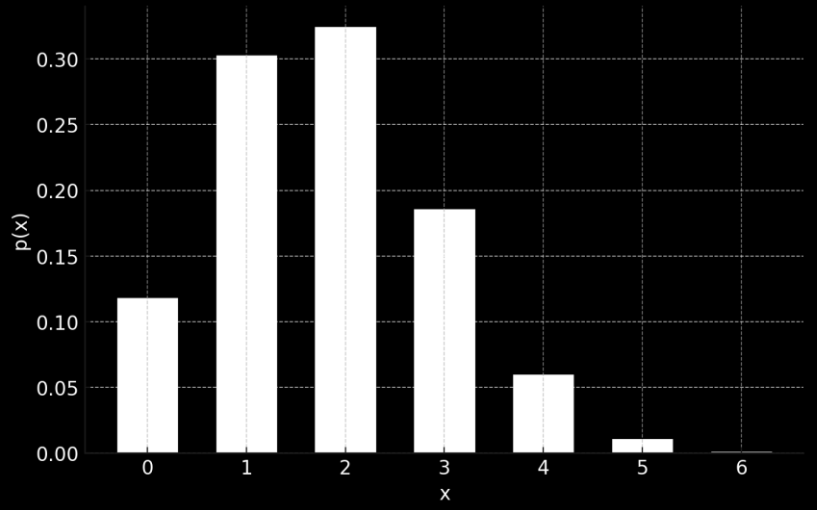
A **discrete random variable** has possible values that can be given in an ordered list.

The **probability distribution** of a discrete random variable X is a list of all possible values of X and their probabilities.

x	0	1	2	3	4	5	6
$p(x)$	0.1176	0.3025	0.3241	0.1852	0.0595	0.0102	0.0007



$$p(x) = \binom{6}{x} 0.3^{\mathrm{x}}(1-0.3)^{6-x}$$

$$for\ x = 0,1,2,3,4,5,6$$

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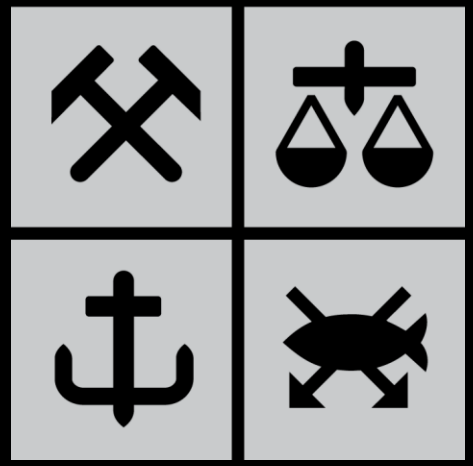

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