Maximum likelihood estimator of μ

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$$\log L(\mu; x_1, \dots, x_n, \sigma^2) = \sum_{i=1}^{n} \log f(x_i; \mu, \sigma^2)$$

$$\boldsymbol{f(x;\ \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$$\log \qquad\qquad\qquad \log$$

$$= \quad -\log\sqrt{2\pi\sigma^2} - \frac{(x-\mu)^2}{2\sigma^2}$$

$$\log\ f(x;\ \mu, \sigma^2)\ =\ -\log\sqrt{2\pi\sigma^2} - \frac{(x-\mu)^2}{2\sigma^2}$$

$$\log L(\mu; x_1, \ldots, x_n, \sigma^2) = \sum_{i=1}^{n} \log f(x_i; \mu, \sigma^2)$$

$$-n\log\sqrt{2\pi\sigma^2} - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= -n\log\sqrt{2\pi\sigma^2} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(x_i^2 - 2\mu x_i + \mu^2\right)$$

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$$\log L(\mu; x_1, \dots, x_n, \sigma^2) = -n \log \sqrt{2\pi\sigma^2} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i^2 - 2\mu x_i + \mu^2)$$

$$\frac{\partial}{\partial \mu} \log L(\mu; x_1, \dots, x_n, \sigma^2) = \underbrace{\frac{\partial}{\partial \mu} \left(-n \log \sqrt{2\pi\sigma^2}\right)}_{=0} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \frac{\partial}{\partial \mu} (x_i^2 - 2\mu x_i + \mu^2)$$

$$= \frac{1}{2\sigma^2} \sum_{i=1}^{n} (-2x_i + 2\mu)$$

$$= \frac{-\sum_{i=1}^{n} x_i + n\mu}{\sigma^2} = 0$$

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$$\frac{-\sum_{i=1}^{n} x_i + n\mu}{\sigma^2} = 0$$

$$-\sum_{i=1}^{n} x_i + n\mu = 0$$

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

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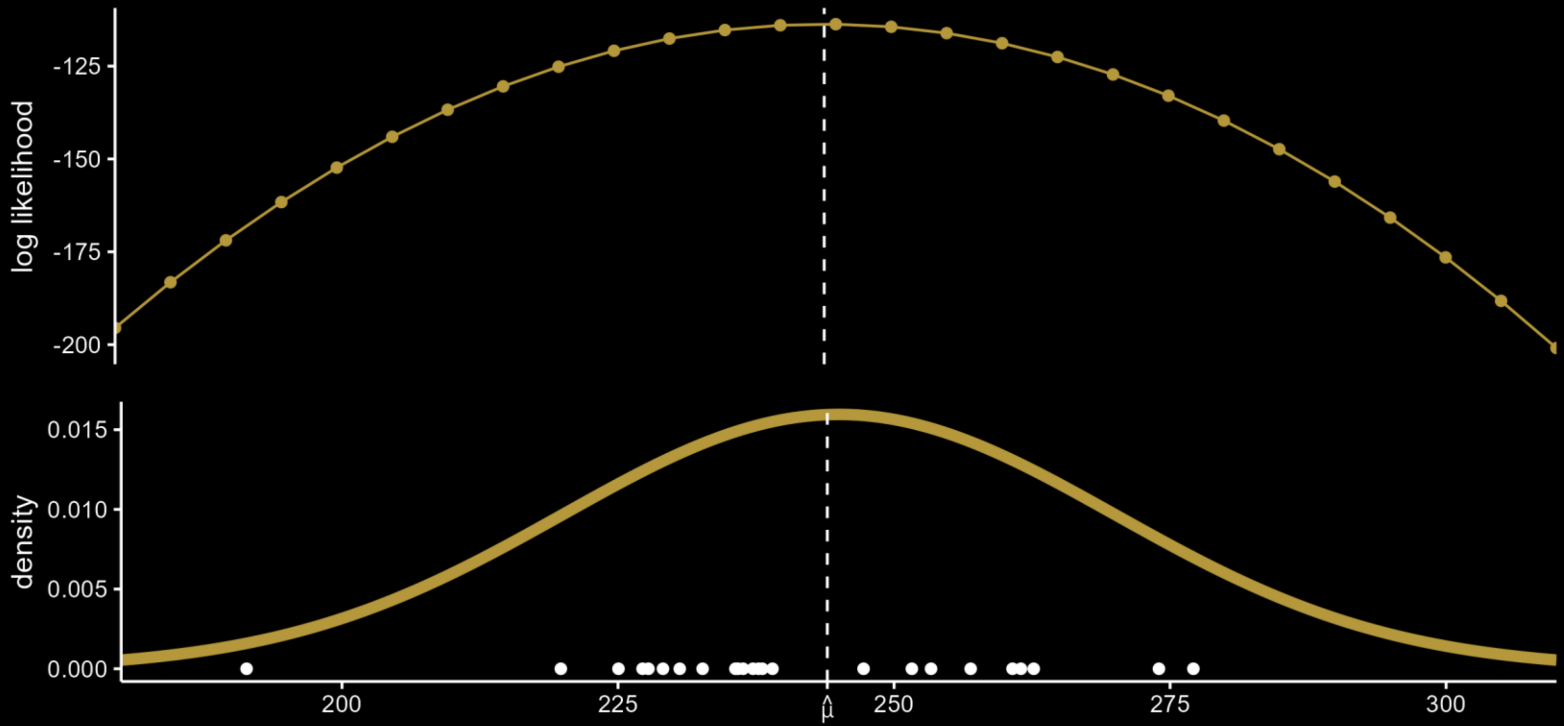

log likelihood
-125
-150
-175
-200
density
0.015
0.010
0.005
0.000
200
225
$\hat{\mu}$
250
275
300

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