Maximum likelihood estimation

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Maximum likelihood estimation

- General estimation procedure
- Parametric distribution assumption
  - Normal, exponential, gamma, binomial, Poisson, etc.
- Finds the parameter value(s) that makes the observed data most probable or likely under the assumed distribution.

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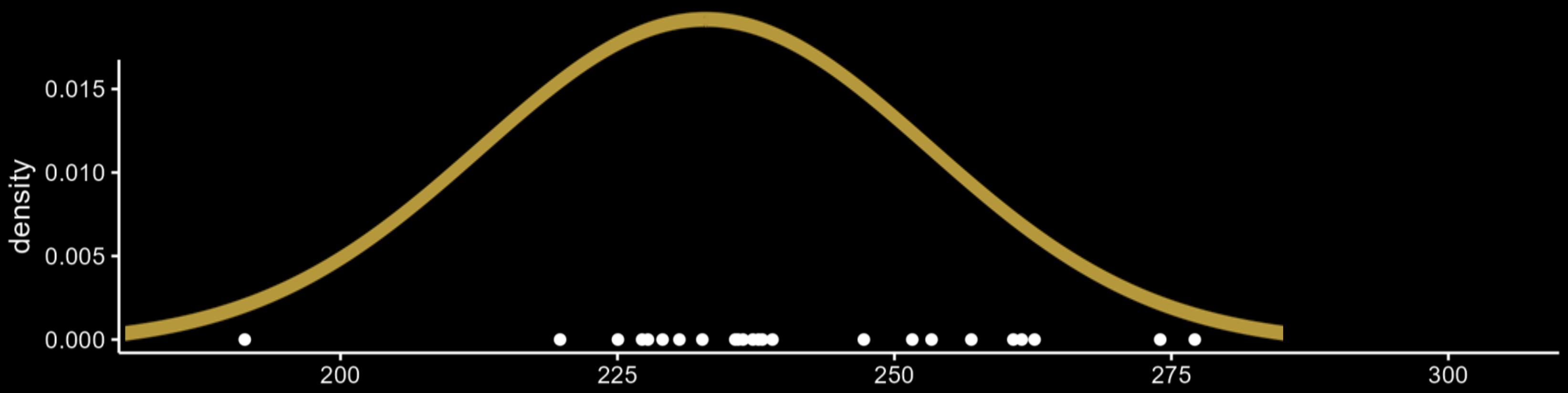
Maximum likelihood estimation

1. Assume a distribution of the observations
2. Express the joint distribution of the observations as as a function of the unknown parameter(s)
   → The likelihood function
3. Find the parameter value(s) that maximizes the likelihood

Step 1

ASSUMPTIONS

- Independent and identically distributed
- Normal distribution
- Known standard deviation $\sigma = 25$
- Unknown mean μ



Step 2

$X_i \sim iid\ N(\mu, \sigma^2)$ for all $i = 1, \dots, n$

$$f(x;\ \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Joint density distribution function:

$$f(x_1, x_2, \dots, x_n;\ \mu, \sigma^2) = f(x_1; \mu, \sigma^2) \cdot f(x_2; \mu, \sigma^2) \cdots f(x_n; \mu, \sigma^2)$$

independence

Identically distributed

$$= \prod_{i=1}^{n} f(x_i; \mu, \sigma^2)$$

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Joint density distribution function:

Step 2

$$f(x_1, x_2, \ldots, x_n;\ \mu, \sigma^2) = \prod_{i=1}^{n} f(x_i; \mu, \sigma^2)$$

The likelihood function:

$$L(\mu; x_1, \ldots, x_n, \sigma^2)$$

Maximum likelihood estimator:

$$\hat{\mu} = \arg\max_{\mu}$$

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The likelihood function

Step 2

$$L(\mu; x_1, \ldots, x_n, \sigma^2) \quad = \quad \prod_{i=1}^{n} f(x_i; \mu, \sigma^2)$$

The log-likelihood function

$$\log \quad = \quad \log$$

$$\sum_{i=1}^{n} \log f(x_i; \mu, \sigma^2)$$

Maximum likelihood estimator:

$$\hat{\mu} = \arg\max_{\mu} \log \mathrm{L}(\mu; x_1, \ldots, x_n, \sigma^2)$$

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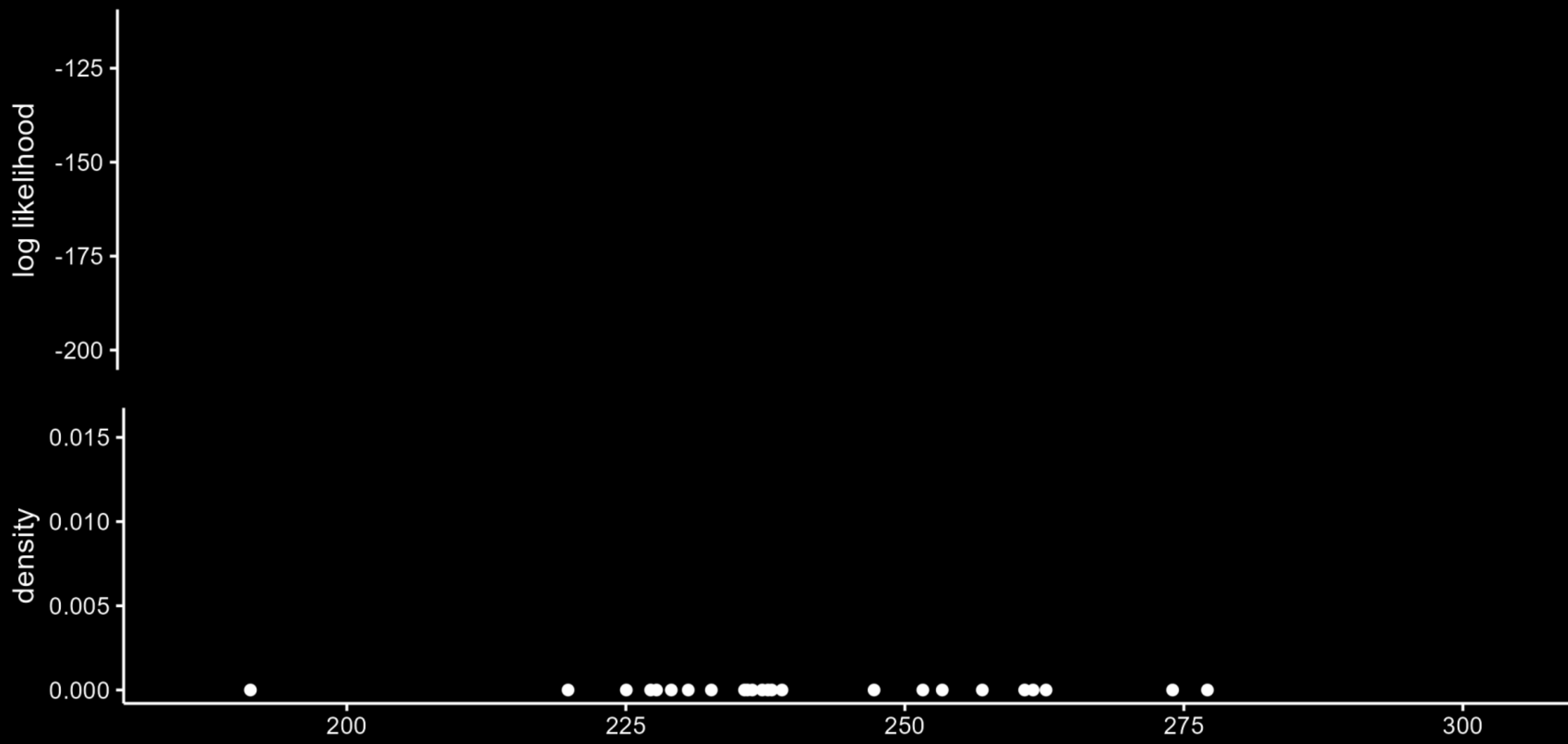
log likelihood
-125
-150
-175
-200
density
0.015
0.010
0.005
0.000
200
225
250
275
300
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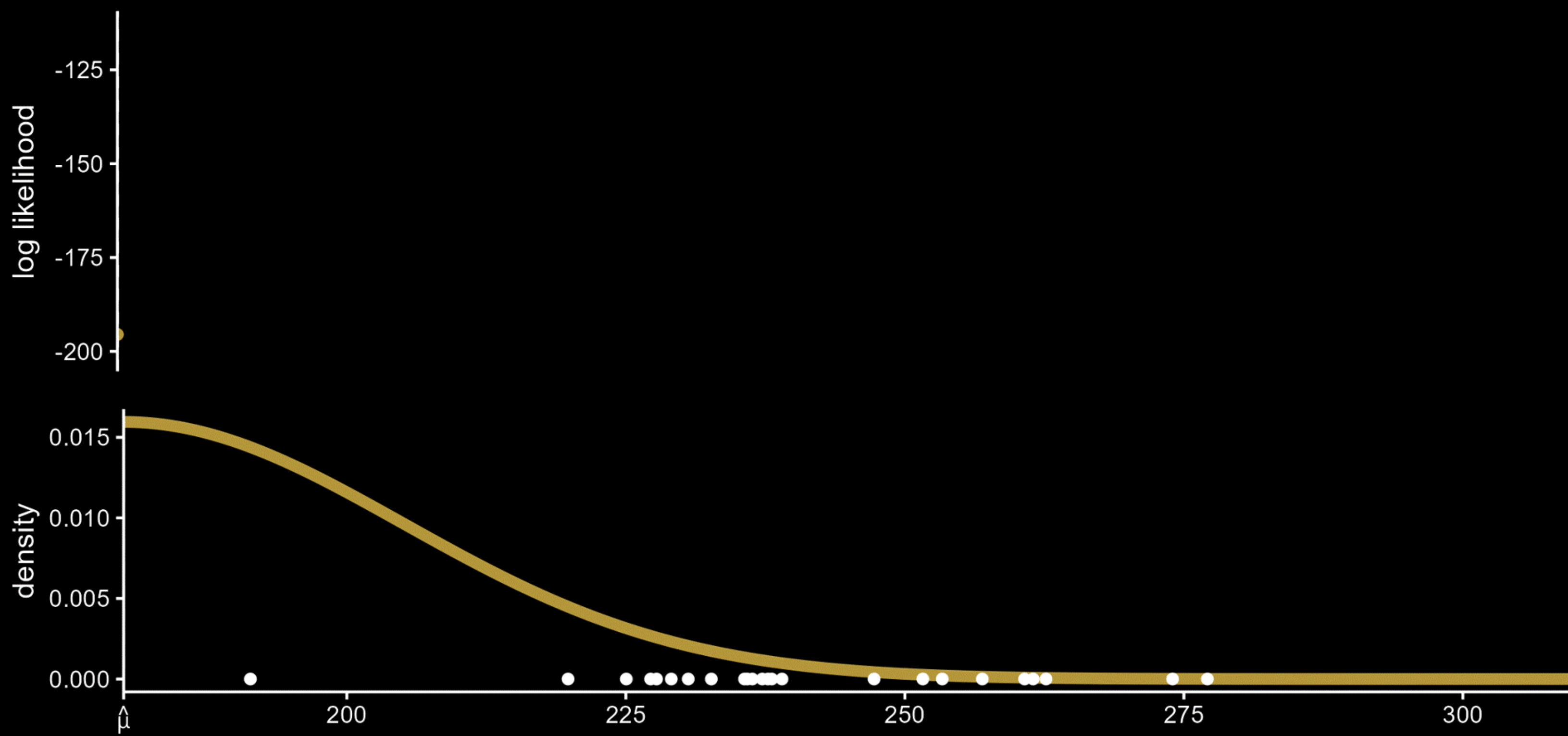
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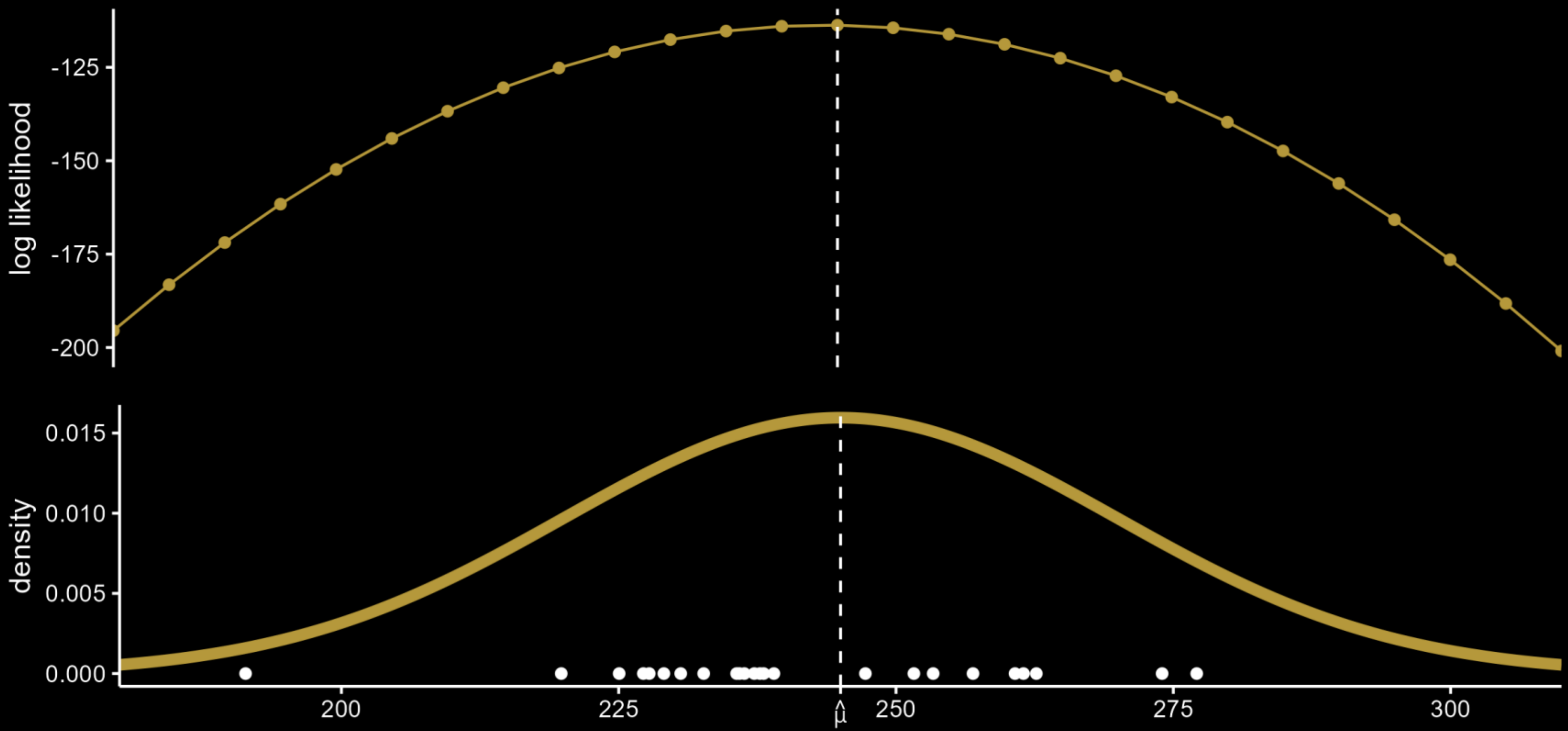

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