Parametric Bootstrap

Parametric bootstrap

- Resampling method
- Variability of an estimator or test statistic
- Simulating data from an assumed parametric model

Assumptions:

- Independent and identically distributed (iid)
- Correct model specification

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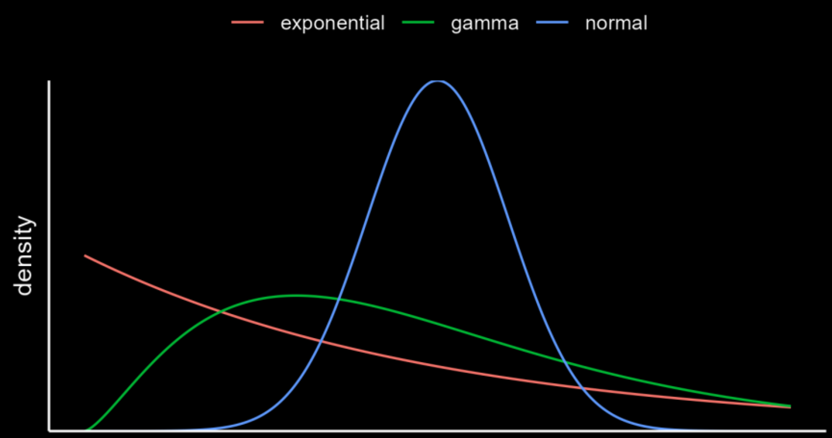
Parametric bootstrap

1. Estimate a parametric distribution function
   - Normal distribution, gamma, exponential, binomial, Poisson etc.
2. Simulate from the estimated distribution
3. Calculate a quantity of interest from each simulated data
4. Aggregate across bootstrap samples



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Estimate bias of the variance estimator

$$X_1, X_2, \dots, X_n \sim iid\ N(\mu, \sigma^2)$$

$$S_n^2 = \frac{1}{n}\sum(X_i - \bar{X})^2$$

$$Bias(S_n^2) = E(S_n^2) - \sigma^2 = \frac{n-1}{n}\sigma^2 - \sigma^2 = -\frac{\sigma^2}{n}$$

iid: Independent and Identically Distributed

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Estimate bias of the variance estimator

- n = 50
- $\mu = 10$
- $\sigma^2 = 4$

$$Bias\left(S_{50}^2\right) = -\frac{\sigma^2}{n} = -\frac{4}{50} = -0.08$$

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Estimate bias of the variance estimator

1. Estimate parameters
2. Simulate from the estimated distribution
3. Calculate a quantity of interest from each simulated data
4. Aggregate across bootstrap samples

$\bar{x}_{50} = 10.21$ and $s_{50}^2 = 3.91$

$$x_{jb} \sim N(10.21, 3.91),$$
$$j = 1, \dots, 50, \;\; b = 1, \dots, 10\,000.$$

$$s_{50,b}^2 = \frac{1}{50}\sum_{j=1}^{50}(x_{jb} - \bar{x}_{50,b})^2,$$

$$\widehat{Bias}(S_{50}^2)$$
$$= \frac{1}{10\,000}\sum s_{50,b}^2 - s_{50}^2$$
$$= 3.82 - 3.91 = -0.085$$

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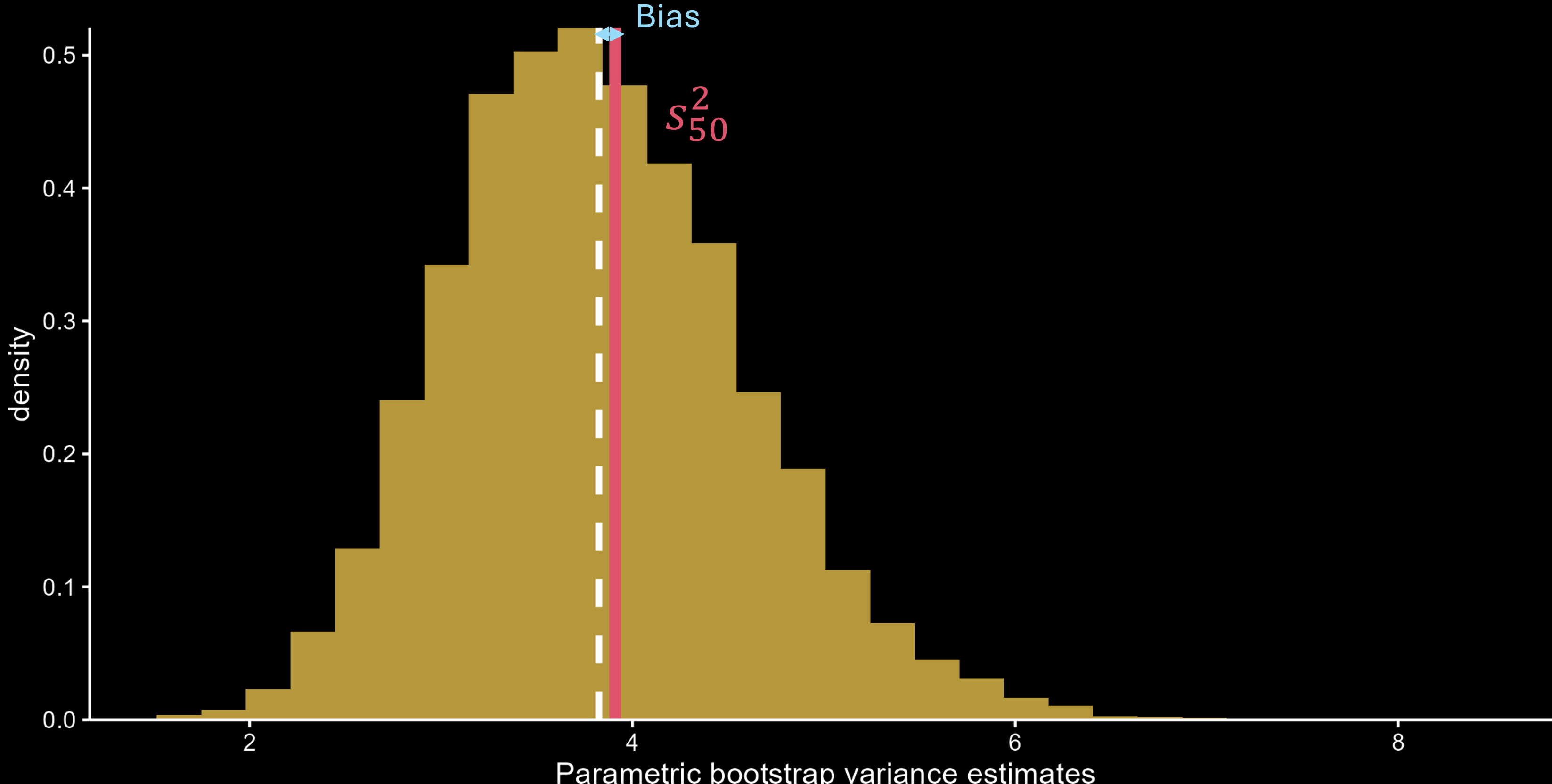
Estimate bias of the variance estimator



Parametric bootstrap or not?

Parametric bootstrap:

- Stronger assumptions on the data generating process
- Depend on quality of parameter estimates

Non-parametric bootstrap:

- Minimal assumptions on the data generating process

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