What is an Estimator?

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An estimator

is a function of the sampled data that provides an estimate of the unknown parameter.

The estimator is the rule/recipe/method.
The estimate is the numerical value obtained after applying the estimator to a given sample.

An estimator is a random variable. (e.g. $\bar{Y}$)
An estimate is not. (e.g. $\bar{y} = 10.2$)

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Two example estimators

Let $Y_1, Y_2, \dots, Y_n$ denote a random sample of independent and identically distributed variables with expectation μ and standard deviation σ.

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \longrightarrow \mu$$

$$S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n} (Y_i - \bar{Y})^2 \longrightarrow \sigma^2$$

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Estimator properties

Let $\hat{\theta}_n$ be an estimator of θ_0.

- Unbiased $E\left(\hat{\theta}_n\right) = \theta_0$
- Consistent $\lim_{n\to\infty} P(|\hat{\theta}_n - \theta_0| > \epsilon) = 0$

$$\lim_{n\to\infty} E\left(\hat{\theta}_n\right) = \theta_0 \text{ and } \lim_{n\to\infty} \mathrm{Var}\left(\hat{\theta}_n\right) = 0$$

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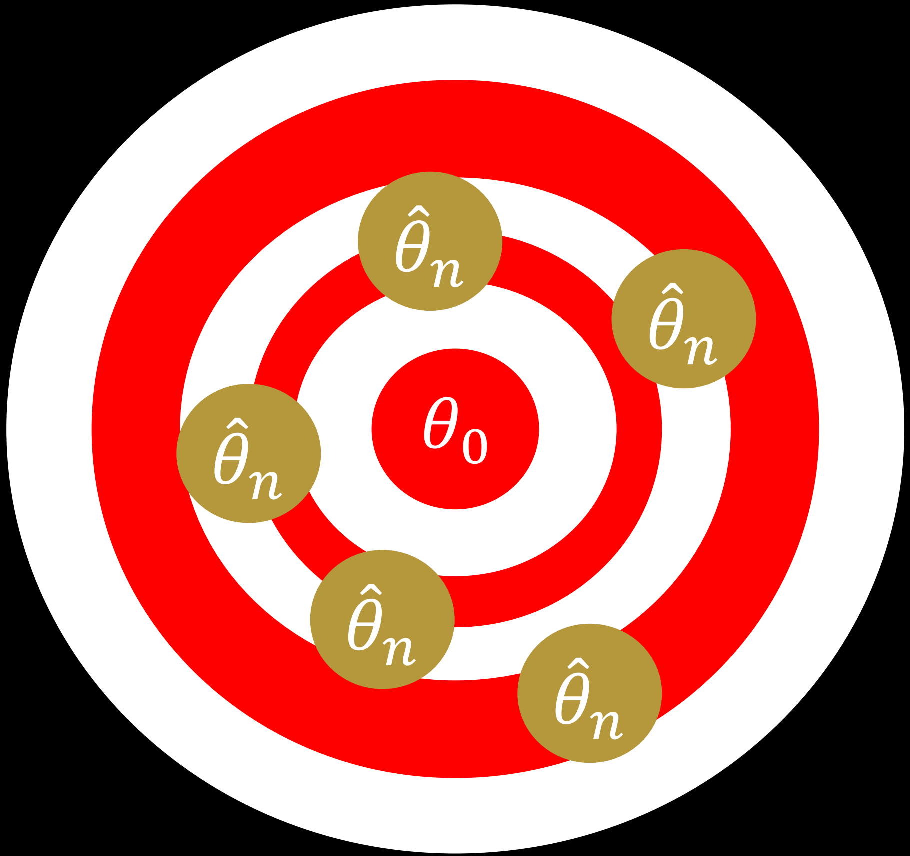
θ_0
$\hat{\theta}_n$
$\hat{\theta}_n$
$\hat{\theta}_n$
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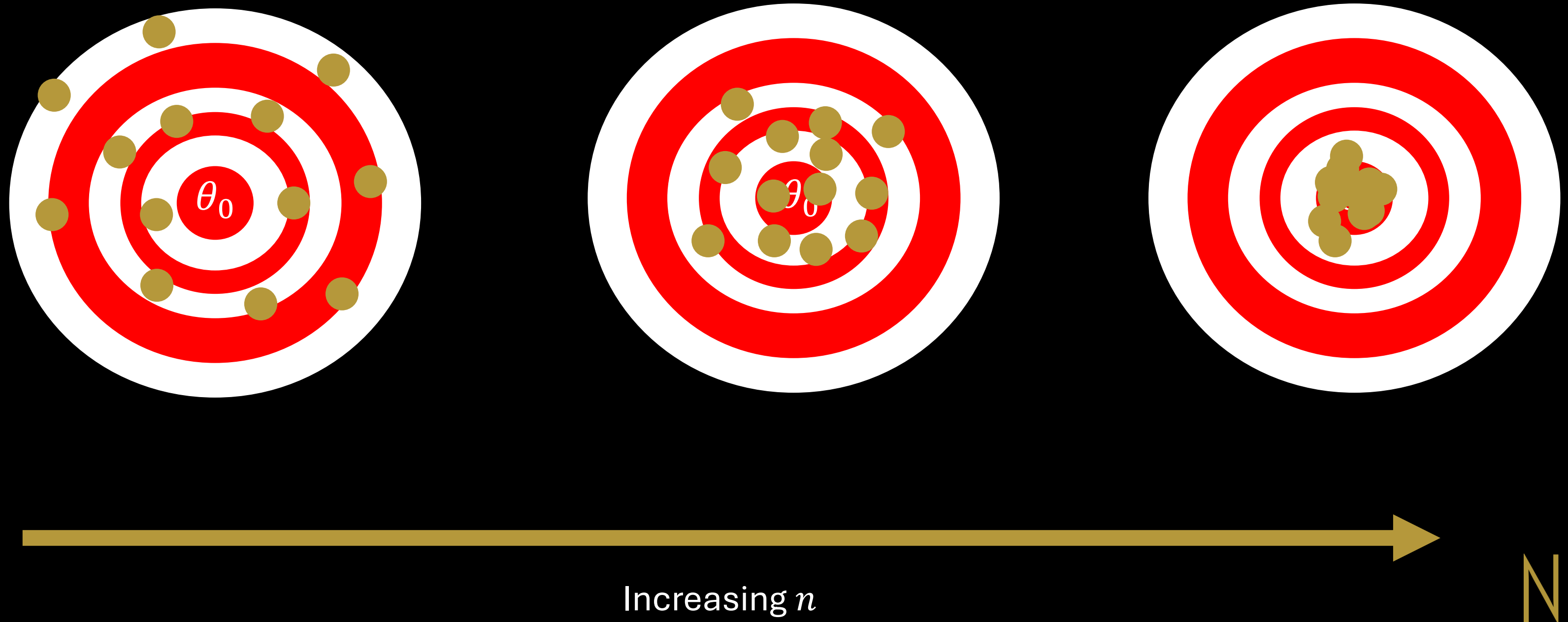
$\hat{\theta}_n$

An unbiased and consistent ESTIMATOR

$\hat{\theta}_n$



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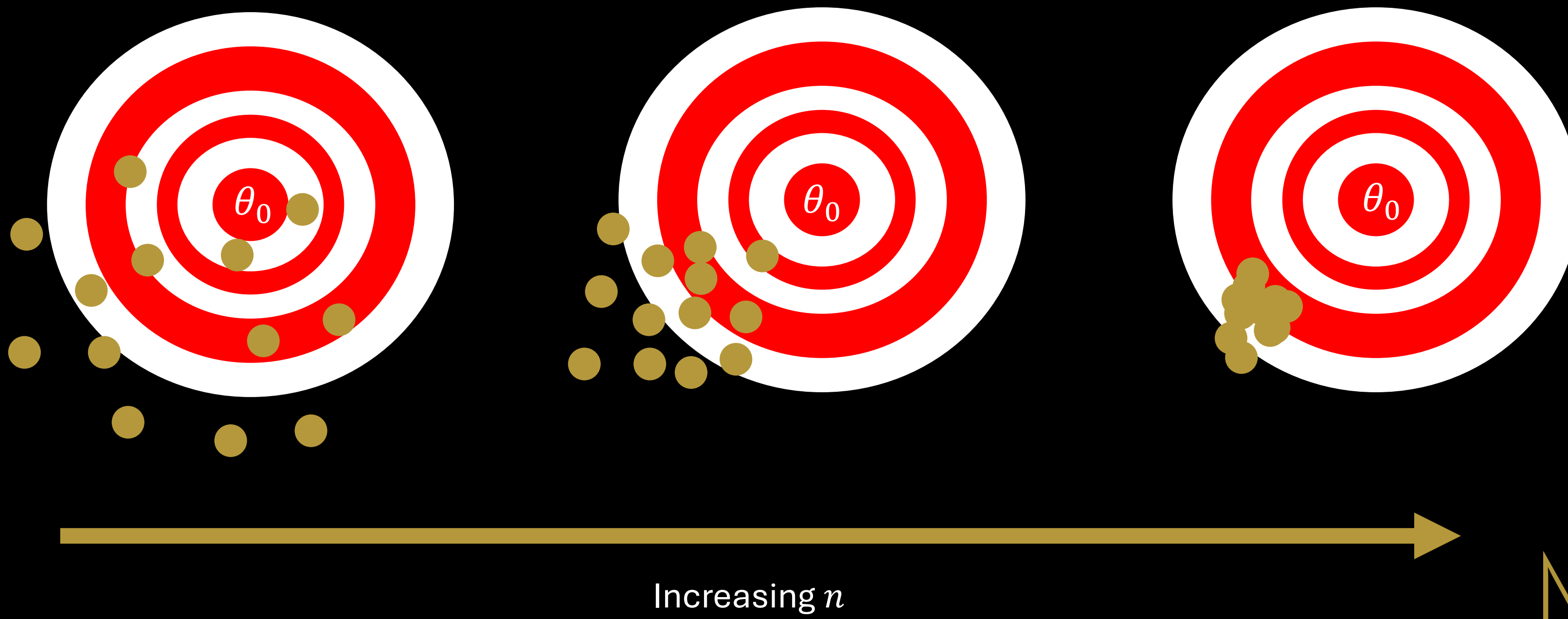
A BIASED ESTIMATOR

$\hat{\theta}_n$



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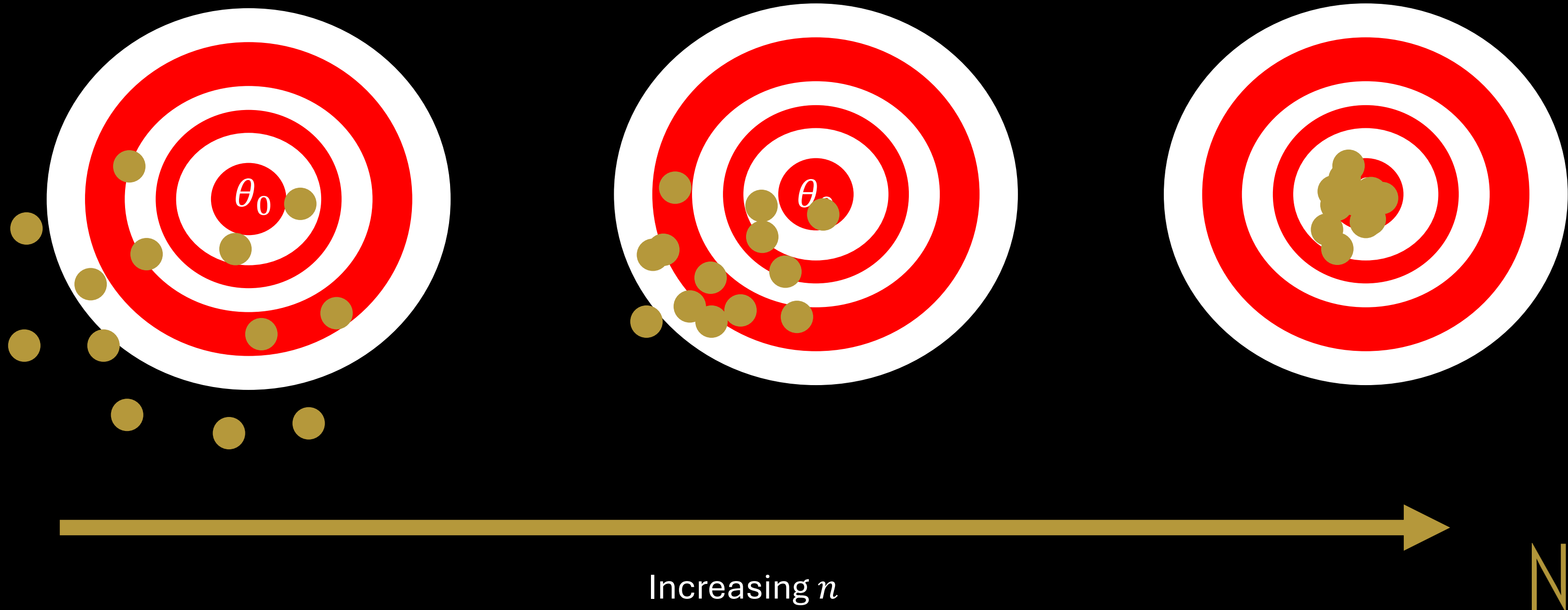
A BIASED, BUT CONSISTENT ESTIMATOR $\hat{\theta}_n$



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The sample mean is unbiased

$$E(\bar{Y}) = E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(Y_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$$

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The sample mean is consistent

$$\mathrm{Var}(\bar{Y}) = \mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) =$$

$$\lim_{n\to\infty} \mathrm{Var}(\bar{Y}) = 0$$

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Variance estimator

As estimator of σ^2, we use

$$S_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2,$$

but why not

$$\frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2?$$

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STANDARD DEVIATION ESTIMATOR

We have that

$$E\,S_Y^2 = E\,\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \sigma^2$$

UNBIASED

while

$$E\,\frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \frac{n-1}{n}\sigma^2 \neq \sigma^2$$

BIASED

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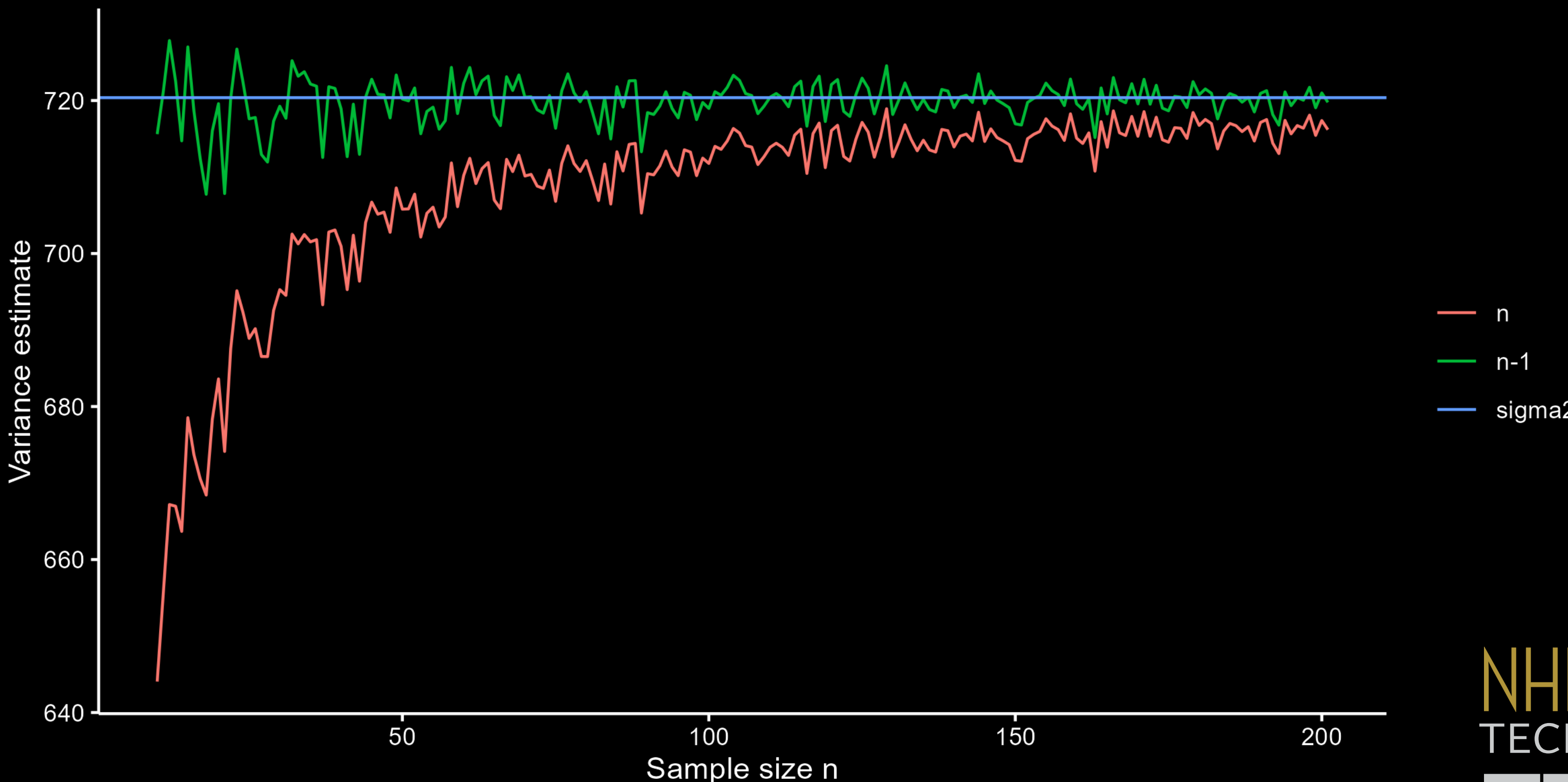

Variance estimate
720
700
680
660
640
50
100
150
200
Sample size n
n
n-1
sigma2

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