Least squares estimators

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$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$Q(\beta_0, \beta_1) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

$$\frac{\partial Q}{\partial \beta_0} = 0 \quad \text{and} \quad \frac{\partial Q}{\partial \beta_1} = 0$$

$$\frac{\partial Q}{\partial \beta_0} = \frac{\partial}{\partial \beta_0} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

$$= \sum_{i=1}^{n} 2(y_i - \beta_0 - \beta_1 x_i) \cdot (-1)$$

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$$= \sum_{i=1}^{n} 2(y_i - \beta_0 - \beta_1 x_i)\cdot(-1)$$

$$= -\cancel{2}\sum y_i + \cancel{2}\beta_0 \cdot n + \cancel{2}\beta_1 \sum x_i = 0$$

$$-\cancel{n}\bar{y} + \cancel{n}\beta_0 + \beta_1 \cancel{n}\bar{x} = 0$$

$$\boxed{\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}}$$

$$\frac{\partial Q}{\partial \beta_1} = \frac{\partial}{\partial \beta_1}\left(\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2\right)$$

$$= \sum_{i=1}^{n} 2(y_i - \beta_0 - \beta_1 x_i)\cdot(-x_i)$$

$$= -\cancel{2}\sum_{i=1}^{n} x_i y_i + \cancel{2}\beta_0 \sum x_i + \cancel{2}\beta_1 \sum^{n} x_i^2 = 0$$

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$$= \sum_{i=1}^{n} 2(y_i - \beta_0 - \beta_1 x_i)\cdot(-x_i)$$

$$= -\cancel{2}\sum_{i=1}^{n} x_i y_i + \cancel{2}\beta_0 \sum x_i + \cancel{2}\beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i - \underbrace{\hat{\beta}_0 \sum_{i=1}^{n} x_i}$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_i = \hat{\beta}_0 n \bar{x}$$

$$= (\bar{y} - \hat{\beta}_1 \bar{x})\, n \bar{x}$$

$$= n \bar{y} \bar{x} - n \hat{\beta}_1 \bar{x}^2$$

$$\hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i - \left(n \bar{x} \bar{y} - n \hat{\beta}_1 \bar{x}^2\right)$$

$$\hat{\beta}_1 \sum_{i=1}^{n} x_i^2 - \hat{\beta}_1 n \bar{x}^2 = \sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - n \bar{x} \bar{y}}{\sum_{i=1}^{n} x_i^2 - n \bar{x}^2}$$

$$\boxed{\begin{aligned} \hat{\beta}_1 &= \frac{\widehat{\mathrm{cov}}(x,y)}{s_x^2} \\ \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} \end{aligned}}$$

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