Logistic regression

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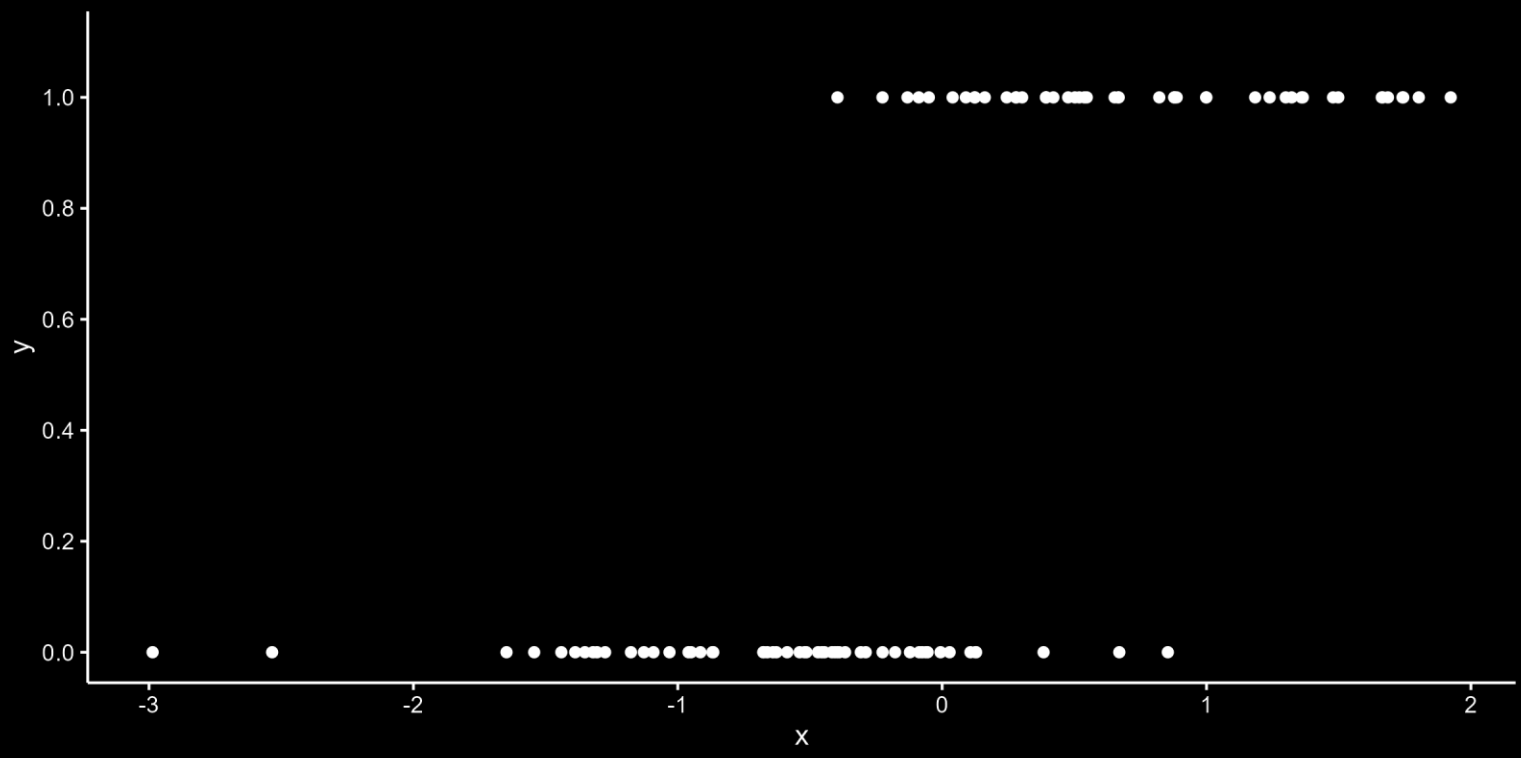
What if Y is a binary variable?



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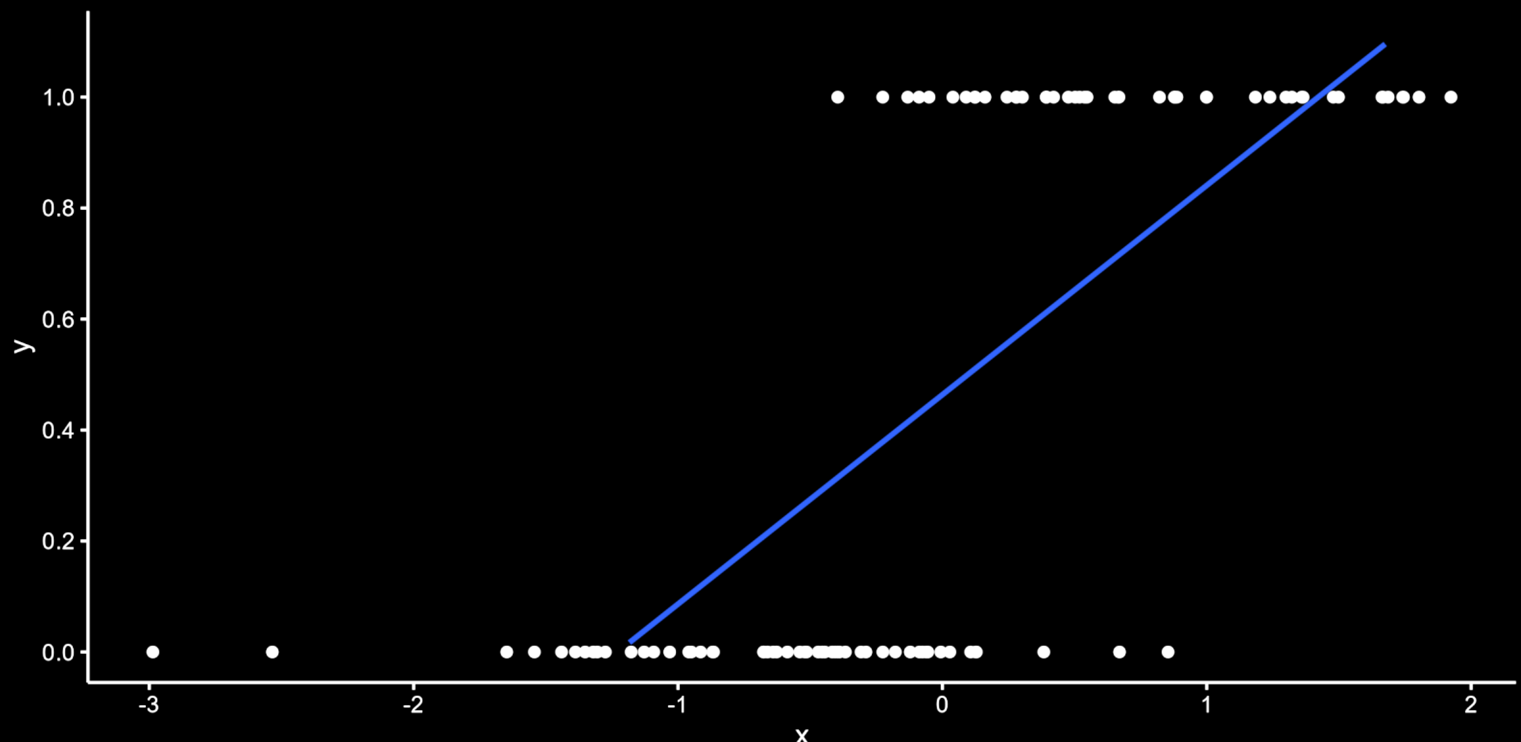

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FROM LINEAR TO LOGISTIC REGRESSION

Instead of modeling Y directly, **logistic regression** models the **probability** that Y=1:

$$P(Y = 1|X = x) \in (0, 1)$$

But we need a way to turn the linear combination of predictors into a value between 0 and 1…

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The logistic function

$$P(Y = 1|x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} \in (0,1)$$

$$P(Y = 1|x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

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$$P(Y=1|x)=\frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}\in(0,1)$$

$$P(Y=0|x)=1-P(Y=1|x)$$

$$=\frac{1}{1+e^{\beta_0+\beta_1 x}}$$

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$$P(Y=1|x) = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}} \in (0,1)$$

$$P(Y=0|x) = \frac{1}{1+e^{\beta_0+\beta_1 x}}$$

$$\frac{P(Y=1|x)}{P(Y=0|x)} = \frac{e^{\beta_0+\beta_1 x}}{\cancel{1+e^{\beta_0+\beta_1 x}}} \frac{\cancel{1+e^{\beta_0+\beta_1 x}}}{1}$$

$$= e^{\beta_0+\beta_1 x}$$

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Linear in the log-odds

$$\log\left(\frac{P(Y=1|x)}{1-P(Y=1|x)}\right) = \beta_0 + \beta_1 x$$

- In linear regression, β_j is the change in Y per unit increase in x_j.
- In logistic regression,
  - β_j is the change in the log-odds of Y per unit increase in x_j.
  - $\exp(\beta_j)$ is the change in the odds ratio of Y per unit increase in x_j.

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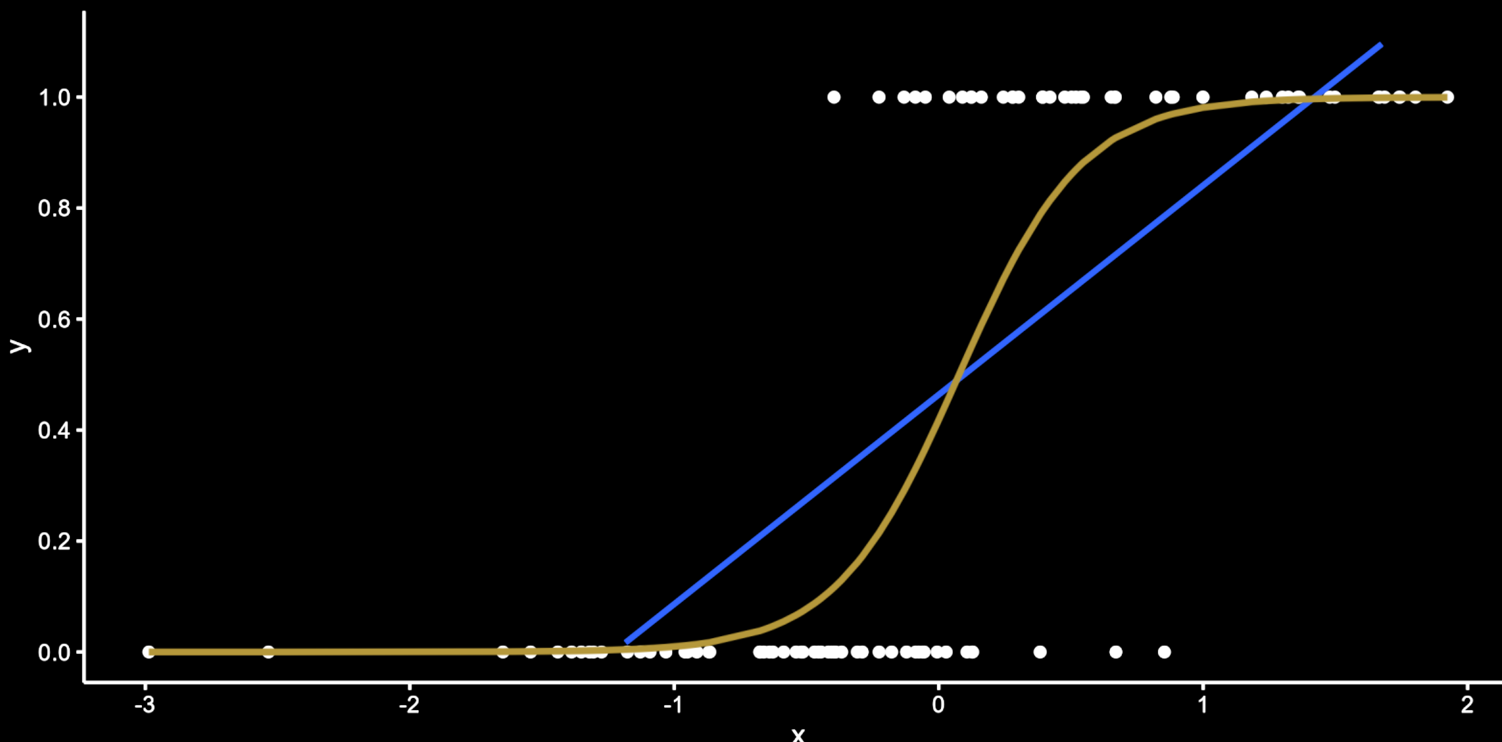

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